North Carolina Department of Public Instruction
INSTRUCTIONAL SUPPORT TOOLS
FOR ACHIEVING NEW STANDARDS

3rd Grade Mathematics ● Unpacked Content

This document is designed to help North Carolina educators teach the Common Core (Standard Course of Study). NCDPI staff are continually updating and improving these tools to better serve teachers. This document was written by the NCDPI Mathematics Consultants with the collaboration of many educators from across the state.

What is the purpose of this document?
To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the CCSS.

What is in the document?
Descriptions of what each standard means a student will know, understand and be able to do. The "unpacking" of the standards done in this document is an effort to answer a simple question "What does this standard mean that a student must know and be able to do?" and to ensure the description is helpful, specific and comprehensive for educators.

How do I send Feedback?
We intend the explanations and examples in this document to be helpful and specific. That said, we believe that as this document is used, teachers and educators will find ways in which the unpacking can be improved and made ever more useful. Please send feedback to us at kitty.rutherford@dpi.mnc.gov or denise.schulz@dpi.nc.gov and we will use your input to refine our unpacking of the standards. Thank You!

Just want the standards alone?
You can find the standards alone at http://corestandards.org/the-standards

Standards for Mathematical Practices

The Common Core State Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

Mathematic Practices	Explanations and Examples
1. Make sense of problems and persevere in solving them.	In third grade, mathematically proficient students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third grade students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" Students listen to other students' strategies and are able to make connections between various methods for a given problem.
2. Reason abstractly and quantitatively.	Mathematically proficient third grade students should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.
3. Construct viable arguments and critique the reasoning of others.	In third grade, mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions that the teacher facilities by asking questions such as "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	Mathematically proficient students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students require extensive opportunities to generate various mathematical representations and to both equations and story problems, and explain connections between representations as well as between representations and equations. Students should be able to use all of these representations as needed. They should evaluate their results in the context of the situation and reflect on whether the results make sense.
5. Use appropriate tools strategically.	Mathematically proficient third grader students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.
6. Attend to precision.	Mathematically proficient third grader students develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.
7. Look for and make use of structure.	In third grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to multiply and divide (commutative and distributive properties).
8. Look for and express regularity in repeated reasoning.	Mathematically proficient students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

Grade 3 Critical Areas

The Critical Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction. The Critical Areas for third grade can be found on page 21 in the *Common Core State Standards for Mathematics*.

1. **Developing understanding of multiplication and division and strategies for multiplication and division within 100.**
   Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

2. **Developing understanding of fractions, especially unit fractions (fractions with numerator 1).**
   Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

3. **Developing understanding of the structure of rectangular arrays and of area**.
   Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

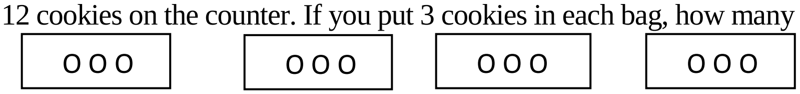
4. **Describing and analyzing two-dimensional shapes**.
   Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Operations and Algebraic Thinking 3.0A

Common Core Cluster

Represent and solve problems involving multiplication and division.
Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **products, groups of, quotients, partitioned equally, multiplication**, **division, equal groups, group size, arrays, equations**, **unknown, expression**

Common Core Standard	**Unpacking** What do these standards mean a child will know and be able to do?
3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	This standard interprets products of whole numbers. Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group or of an equal amount of objects were added or collected numerous times. Multiplication requires students to think in terms of groups of things rather than individual things. Students learn that the multiplication symbol 'x' means "groups of" and problems such as 5 x 7 refer to 5 groups of 7. Example: Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase? 5 groups of 3, 5 x 3 = 15. Describe another situation where there would be 5 groups of 3 or 5 x 3. Sonya earns $7 a week pulling weeds. After 5 weeks of work, how much has Sonya worked? Write an equation and find the answer. Describe another situation that would match 7x5.
3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	This standard focuses on two distinct models of division: partition models and measurement (repeated subtraction) models. Partition models provide students with a total number and the number of groups. These models focus on the question, "How many objects are in each group so that the groups are equal?" A context for partition models would be: There are 12 cookies on the counter. If you are sharing the cookies equally among three bags, how many cookies will go in each bag? Measurement (repeated subtraction) models provide students with a total number and the number of objects in each group. These models focus on the question, "How many equal groups can you make?" A context for measurement models would be: There are 12 cookies on the counter. If you put 3 cookies in each bag, how many bags will you fill? 

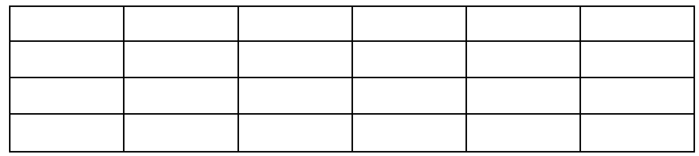
3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

[1] See Glossary, Table 2. (page 89) (Table included at the end of this document for your convenience)

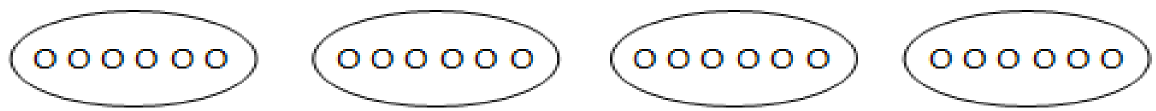
This standard references various problem solving context and strategies that students are expected to use while solving word problems involving multiplication & division. Students should use a variety of representations for creating and solving one-step word problems, such as: If you divide 4 packs of 9 brownies among 6 people, how many brownies does each person receive? *(4 x 9 = 36, 36 ÷ 6 = 6).*

Glossary page 89, Table 2 (table also included at the end of this document for your convenience) gives examples of a variety of problem solving contexts, in which students need to find the product, the group size, or the number of groups. Students should be given ample experiences to explore all of the different problem structures.

Examples of multiplication:
There are 24 desks in the classroom. If the teacher puts 6 desks in each row, how many rows are there?
This task can be solved by drawing an array by putting 6 desks in each row. This is an array model.



This task can also be solved by drawing pictures of equal groups.
4 groups of 6 equals 24 objects



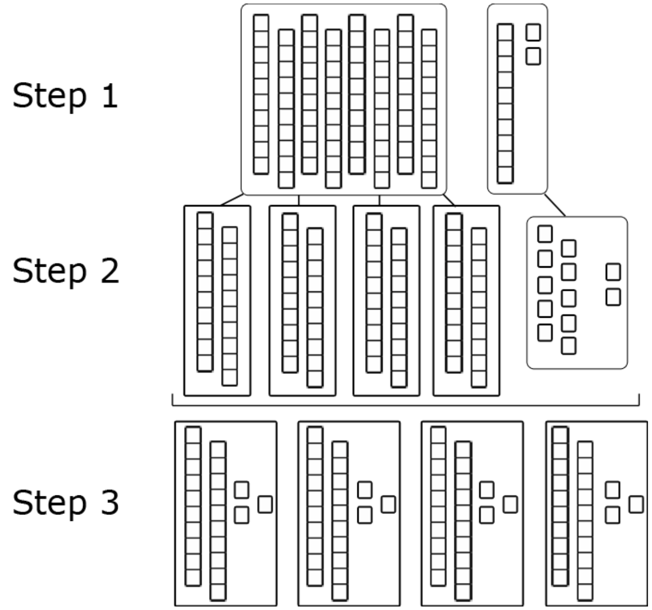
A student can also reason through the problem mentally or verbally, "I know 6 and 6 are 12. 12 and 12 are 24. Therefore, there are 4 groups of 6 giving a total of 24 desks in the classroom."

A number line could also be used to show equal jumps.

Students in third grade should use a variety of pictures, such as stars, boxes, flowers to represent unknown numbers (variables). Letters are also introduced to represent unknowns in third grade.

Examples of Division:
There are some students at recess. The teacher divides the class into 4 lines with 6 students in each line. Write a division equation for this story and determine how many students are in the class *(☐ ÷ 4*
students in the class).

Determining the number of objects in each share (partition model of division, where the size of the groups is unknown):
Example:
The bag has 92 hair clips, and Laura and her three friends want to share them equally. How many hair clips will each person receive?



Determining the number of shares (measurement division, where the number of groups is unknown)
Example:
Max the monkey loves bananas. Molly, his trainer, has 24 bananas. If she gives Max 4 bananas each day, how many days will the bananas last?

Starting	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
24	24 – 4 = 20	20 – 4 = 16	16 – 4 = 12	12 – 4 = 8	8 – 4 = 4	4 – 4 = 0

Solution: The bananas will last for 6 days.

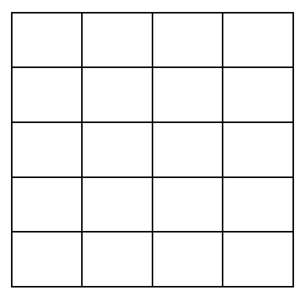
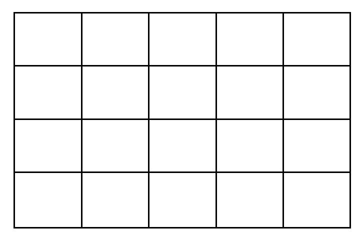
3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?*	This standard refers to Glossary page 89, Table 2 (table also included at the end of this document for your convenience) and equations for the different types of multiplication and division problem structures. The easiest problem structure includes Unknown Product (3 x 6 = ? or 18 ÷ 3 = 6). The more difficult problem structures include Group Size Unknown (3 x ? = 18 or 18 ÷ 3 = 6) or Number of Groups Unknown (? x 6 = 18, 18 ÷ 6 = 3). The focus of 3.OA.4 extends beyond the traditional notion of *fact families*, by having students explore the inverse relationship of multiplication and division. Students extend work from earlier grades with their understanding of the meaning of the equal sign as “the same amount as” to interpret an equation with an unknown. When given 4 x ? = 40, they might think: • 4 groups of some number is the same as 40 • 4 times some number is the same as 40 • I know that 4 groups of 10 is 40 so the unknown number is 10 • The missing factor is 10 because 4 times 10 equals 40. Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions. Example: Solve the equations below: 24 = ? x 6 72 ÷ △ = 9 Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether? 3 x 4 = *m*

Common Core Cluster

Understand properties of multiplication and the relationship between multiplication and division.

Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

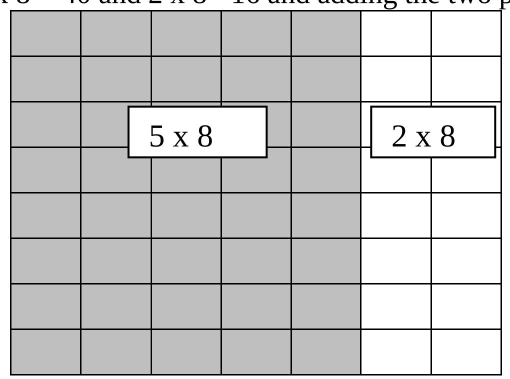
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **operation, multiply, divide, factor, product, quotient, unknown, strategies, (properties)-rules about how numbers work**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
3.OA.5 Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* [2] Students need not use formal terms for these properties.	This standard references properties (rules about how numbers work) of multiplication. This extends past previous expectations, in which students were asked to identify properties. While students DO NOT need to not use the formal terms of these properties, student must understand that properties are rules about how numbers work, and they need to be flexibly and fluently applying each of them in various situations. Students represent expressions using various objects, pictures, words and symbols in order to develop their understanding of properties. They multiply by 1 and 0 and divide by 1. They change the order of numbers to determine that the order of numbers does not make a difference in multiplication (but does make a difference in division). Given three factors, they investigate changing the order of how they multiply the numbers to determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication. The associative property states that the sum or product stays the same when the grouping of addends or factors is changed. For example, when a student multiplies 7 x 5 x 2, a student could rearrange the numbers to first multiply 5 x 2 = 10 and then multiply 10 x 7 = 70. The commutative property (order property) states that the order of numbers does not matter when you are adding or multiplying numbers. For example, if a student knows that 5 x 4 = 20, then they also know that 4 x 5 = 20. The array below could be described as a 5 x 4 array for 5 columns and 4 rows, or a 4 x 5 array for 4 rows and 5 columns. There is no "fixed" way to write the dimensions of an array as rows x columns or columns x rows. Students should have flexibility in being able to describe both dimensions of an array. Example: 4 x 5 or 5 x 4   4 x 5 or 5 x 4

Students are introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they don't know. Students would be using mental math to determine a product. Here are ways that students could use the distributive property to determine the product of 7 x 6. Again, students should use the distributive property, but can refer to this in informal language such as "breaking numbers apart".

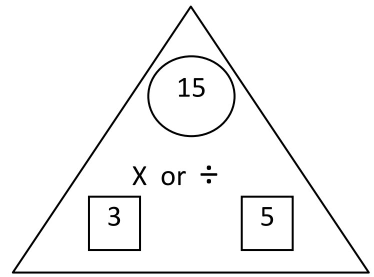
Student 1	Student 2	Student 3
7 x 6	7 x 6	7 x 6
7 x 5 = 35	7 x 3 = 21	5 x 6 = 30
7 x 1 = 7	7 x 3 = 21	2 x 6 = 12
35 + 7 = 42	21 + 21 = 42	30 + 12 = 42

Another example if the distributive property helps students determine the products and factors of problems by breaking numbers apart. For example, for the problem 7 x 8 = ?, students can decompose the 7 into a 5 and 2, and reach the answer by multiplying 5 x 8 = 40 and 2 x 8 =16 and adding the two products (40 +16 = 56).



To further develop understanding of properties related to multiplication and division, students use different representations and their understanding of the relationship between multiplication and division to determine if the following types of equations are true or false.

- 0 x 7 = 7 x 0 = 0 (Zero Property of Multiplication)
- 1 x 9 = 9 x 1 = 9 (Multiplicative Identity Property of 1)
- 3 x 6 = 6 x 3 (Commutative Property)
- 8 ÷ 2 = 2 ÷ 8 (Students are only to determine that these are not equal)
- 2 x 3 x 5 = 6 x 5
- 10 x 2 < 5 x 2 x 2
- 2 x 3 x 5 = 10 x 3
- 0 x 6 > 3 x 0 x 2

3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*	This standard refers the Glossary on page 89, Table 2 (table also included at the end of this document for your convenience) and the various problem structures. Since multiplication and division are inverse operations, students are expected to solve problems and explain their processes of solving division problems that can also be represented as unknown factor multiplication problems. Example: A student knows that 2 x 9 = 18. How can they use that fact to determine the answer to the following question: 18 people are divided into pairs in P.E. class? How many pairs are there? Write a division equation and explain your reasoning. Multiplication and division are inverse operations and that understanding can be used to find the unknown. Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient. Examples: • 3 x 5 = 15 5 x 3 = 15 • 15 ÷ 3 = 5 15 ÷ 5 = 3  Example: Sarah did not know the answer to 63 divided by 7. Are each of the following was an appropriate way for Sarah to think about the problem? Explain why or why not with a picture or words for each one. • “I know that 7 x 9 = 63, so 63 divided by 7 must be 9.” • “I know that 7x10 = 70. If I take away a group of 7, that means that I have 7x9 = 63. So 63 divided by 7 is 9.” • “I know that 7x5 is 35. 63 minus 35 is 28. I know that 7x4 = 28. So if I add 7x5 and 7x4 I get 63. That means that 7x9 is 63, or 63 divided by 7 is 9.”

Common Core Cluster	
Multiply and divide within 100.	
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **operation, multiply, divide, factor, product, quotient, unknown, strategies, reasonableness, mental computation, property**	
Common Core Standard	**Unpacking** What do these standards mean a child will know and be able to do?
3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	This standard uses the word fluently, which means with accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property). "Know from memory" should not focus only on timed tests and repetitive practice, but ample experiences working with manipulatives, pictures, arrays, word problems, and numbers to internalize the basic facts (up to 9 x 9). By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts through 10 and the related division facts. Multiplying and dividing fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Strategies students may use to attain fluency include: • Multiplication by zeros and ones • Doubles (2s facts), Doubling twice (4s), Doubling three times (8s) • Tens facts (relating to place value, 5 x 10 is 5 tens or 50) • Five facts (half of tens) • Skip counting (counting groups of __ and knowing how many groups have been counted) • Square numbers (ex: 3 x 3) • Nines (10 groups less one group, e.g., 9 x 3 is 10 groups of 3 minus one group of 3) • Decomposing into known facts (6 x 7 is 6 x 6 plus one more group of 6) • Turn-around facts (Commutative Property) • Fact families (Ex: 6 x 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 x 6 = 24) • Missing factors Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms.

	Note that mastering this material, and reaching fluency in single-digit multiplications and related divisions with understanding, may be quite time consuming because there are no general strategies for multiplying or dividing all single-digit numbers as there are for addition and subtraction. Instead, there are many patterns and strategies dependent upon specific numbers. So it is imperative that extra time and support be provided if needed. (*Progressions for the CCSSM; Operations and Algebraic Thinking*, CCSS Writing Team, May 2011, page 22) All of the understandings of multiplication and division situations (See Glossary, Table 2. (page 89) Table included at the end of this document for your convenience), of the levels of representation and solving, and of patterns need to culminate by the end of Grade 3 in fluent multiplying and dividing of all single-digit numbers and 10. Such fluency may be reached by becoming fluent for each number (e.g., the 2s, the 5s, etc.) and then extending the fluency to several, then all numbers mixed together. Organizing practice so that it focuses most heavily on understood but not yet fluent products and unknown factors can speed learning. To achieve this by the end of Grade 3, students must begin working toward fluency for the easy numbers as early as possible. Because an unknown factor (a division) can be found from the related multiplication, the emphasis at the end of the year is on knowing from memory all products of two one-digit numbers. As should be clear from the foregoing, this isn't a matter of instilling facts divorced from their meanings, but rather the outcome of a carefully designed learning process that heavily involves the interplay of practice and reasoning. All of the work on how different numbers fit with the base-ten numbers culminates in these "just know" products and is necessary for learning products. Fluent dividing for all single-digit numbers, which will combine just knows, knowing from a multiplication, patterns, and best strategy, is also part of this vital standard. (*Progressions for the CCSSM; Operations and Algebraic Thinking*, CCSS Writing Team, May 2011, page 27)

Common Core Cluster

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **operation, multiply, divide, factor, product, quotient, subtract, add, addend, sum, difference, equation, expression, unknown, strategies, reasonableness, mental computation, estimation, rounding, patterns, (properties)-rules about how numbers work, input and output table**

<table>
<tr><th>Common Core Standard</th><th>Unpacking
What do these standards mean a child will know and be able to do?</th></tr>
<tr><td>3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3]

[3] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order.</td><td>Students in third grade begin the step to formal algebraic language by using a letter for the unknown quantity in expressions or equations for one and two-step problems. But the symbols of arithmetic, x or • or * for multiplication and ÷ or / for division, continue to be used in Grades 3, 4, and 5. (Progressions for the CCSSM; Operations and Algebraic Thinking, CCSS Writing Team, May 2011, page 27)

This standard refers to two-step word problems using the four operations. The size of the numbers should be limited to related 3rd grade standards (e.g., 3.OA.7 and 3.NBT.2). Adding and subtracting numbers should include numbers within 1,000, and multiplying and dividing numbers should include single-digit factors and products less than 100.

This standard calls for students to represent problems using equations with a letter to represent unknown quantities.

Example:
Mike runs 2 miles a day. His goal is to run 25 miles. After 5 days, how many miles does Mike have left to run in order to meet his goal? Write an equation and find the solution (2 x 5 + m = 25).

This standard refers to estimation strategies, including using compatible numbers (numbers that sum to 10, 50, or 100) or rounding. The focus in this standard is to have students use and discuss various strategies. Students should estimate during problem solving, and then revisit their estimate to check for reasonableness.

Example:
Here are some typical estimation strategies for the problem:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel?
<table>
<tr><td>Student 1
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.</td><td>Student 2
I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.</td><td>Student 3
I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530.</td></tr>
</table></td></tr>
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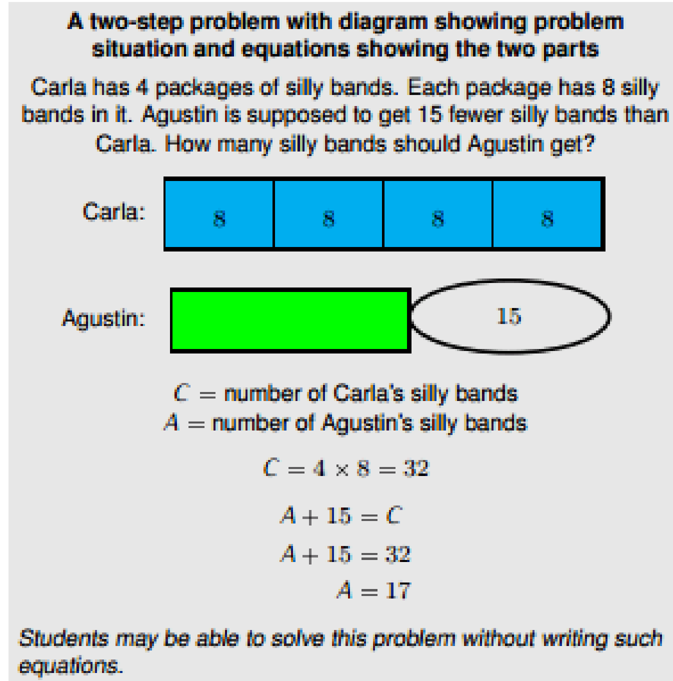
The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer.



(*Progressions for the CCSSM; Operations and Algebraic Thinking,* CCSS Writing Team, May 2011, page 28)

In the diagram above, Carla's bands are shown using 4 equal-sized bars that represent 4x8 or 32 bands. Agustin's bands are directly below showing that the number that August in has plus 15 = 32. The diagram can also be drawn like this:

8	8	8	8
15		?	

3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.
For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

This standard calls for students to examine arithmetic patterns involving both addition and multiplication. Arithmetic patterns are patterns that change by the same rate, such as adding the same number. For example, the series 2, 4, 6, 8, 10 is an arithmetic pattern that increases by 2 between each term.

This standard also mentions identifying patterns related to the properties of operations.
Examples:

- Even numbers are always divisible by 2. Even numbers can always be decomposed into 2 equal addends (14 = 7 + 7).
- Multiples of even numbers (2, 4, 6, and 8) are always even numbers.
- On a multiplication chart, the products in each row and column increase by the same amount (skip counting).
- On an addition chart, the sums in each row and column increase by the same amount.

What do you notice about the numbers highlighted in pink in the multiplication table?
Explain a pattern using properties of operations.
When (commutative property) one changes the order of the factors they will still gets the same product, example 6 x 5 = 30 and 5 x 6 = 30.

x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Teacher: What pattern do you notice when 2, 4, 6, 8, or 10 are multiplied by any number (even or odd)?
Student: The product will always be an even number.
Teacher: Why?

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

What patterns do you notice in this addition table? Explain why the pattern works this way?

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	19	11	12	13	14	15	16	17	18	19	20

Students need ample opportunities to observe and identify important numerical patterns related to operations. They should build on their previous experiences with properties related to addition and subtraction. Students investigate addition and multiplication tables in search of patterns and explain why these patterns make sense mathematically. Example:

- Any sum of two even numbers is even.
- Any sum of two odd numbers is even.
- Any sum of an even number and an odd number is odd.
- The multiples of 4, 6, 8, and 10 are all even because they can all be decomposed into two equal groups.
- The doubles (2 addends the same) in an addition table fall on a diagonal while the doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines.
- The multiples of any number fall on a horizontal and a vertical line due to the commutative property.
- All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10.

Students also investigate a hundreds chart in search of addition and subtraction patterns. They record and organize all the different possible sums of a number and explain why the pattern makes sense.

addend	addend	sum
0	20	20
1	19	20
2	18	20
3	17	20
4	16	20
□	□	□
□	□	□
□	□	□
20	0	20

Number and Operations in Base Ten — 3.NBT

Common Core Cluster

Use place value understanding and properties of operations to perform multi-digit arithmetic.[1]
[1] A range of algorithms may be used.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, round, addition, add, addend, sum, subtraction, subtract, difference, strategies, (properties)-rules about how numbers work**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.	This standard refers to place value understanding, which extends beyond an algorithm or memorized procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding. Mrs. Rutherford drives 158 miles on Saturday and 171 miles on Sunday. When she told her husband she estimated how many miles to the nearest 10 before adding the total. When she told her sister she estimated to the nearest 100 before adding the total. Which method provided a closer estimate?
3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. [1] A range of algorithms may be used.	This standard refers to fluently, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property). The word algorithm refers to a procedure or a series of steps. There are other algorithms other than the standard algorithm. Third grade students should have experiences beyond the standard algorithm. A variety of algorithms and strategies will be assessed on North Carolina EOG assessment. Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties. Students explain their thinking and show their work by using strategies and algorithms, and verify that their answer is reasonable. **Computation algorithm.** A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. **Computation strategy.** Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. (*Progressions for the CCSSM; Number and Operation in Base Ten,* CCSS Writing Team, April 2011, page 2) Example:

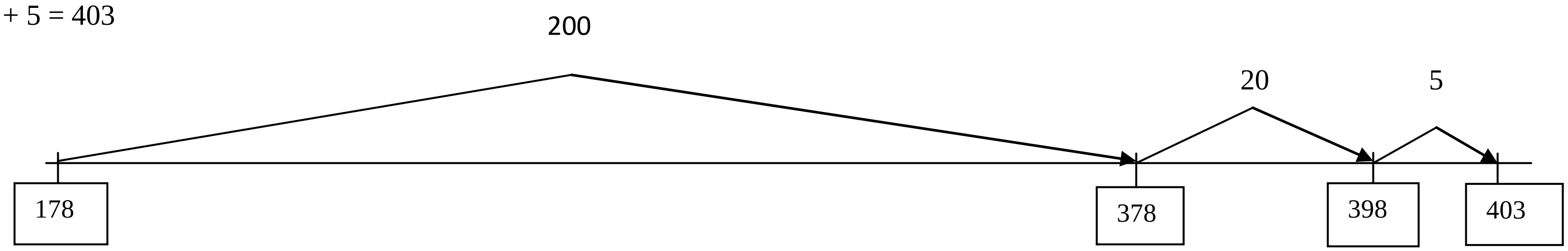
There are 178 fourth graders and 225 fifth graders on the playground. What is the total number of students on the playground?

Student 1
100 + 200 = 300
70 + 20 = 90
8 + 5 = 13
300 + 90 + 13 = 403 students

Student 2
I added 2 to 178 to get 180. I added 220 to get 400. I added the 3 left over to get 403.

Student 3
I know the 75 plus 25 equals 100. I then added 1 hundred from 178 and 2 hundreds from 275. I had a total of 4 hundreds and I had 3 more left to add. So I have 4 hundreds plus 3 more which is 403.

Student 4
178 + 225 = ?
178 + 200 = 378
378 + 20 = 398
398 + 5 = 403



3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.	This standard extends students' work in multiplication by having them apply their understanding of place value. This standard expects that students go beyond tricks that hinder understanding such as "just adding zeros" and explain and reason about their products. For example, for the problem 50 x 4, students should think of this as 4 groups of 5 tens or 20 tens, and twenty tens equals 200. The special role of 10 in the base-ten system is important in understanding multiplication of one-digit numbers with multiples of 10. For example, the product 3 x 50 can be represented as 3 groups of 5 tens, which is 15 tens, which is 150. This reasoning relies on the associative property of multiplication: 3 x 50 = 3 x (5 x 10) = (3 x 5) x 10 = 15 x 10 = 150. It is an example of how to explain an instance of a calculation pattern for these products: calculate the product of the non-zero digits, and then shift the product one place to the left to make the result ten times as large • • **Grade 3 explanations for "15 tens is 150"** • Skip-counting by 50. 5 tens is 50, 100, 150. • Counting on by 5 tens. 5 tens is 50, 5 more tens is 100, 5 more tens is 150. • Decomposing 15 tens. 15 tens is 10 tens and 5 tens. 10 tens is 100. 5 tens is 50. So 15 tens is 100 and 50, or 150. • Decomposing 15. $15 \times 10 = (10+5) \times 10 = (10 \times 10) + (5 \times 10) = 100 + 50 = 150$ *All of these explanations are correct. However, skip-counting and counting on become more difficult to use accurately as numbers become larger, e.g., in computing* 5×90 *or explaining why* 45 *tens is* 450*, and needs modification for products such as* 4×90*. The first does not indicate any place value understanding.* (*Progressions for the CCSSM; Number and Operation in Base Ten*, CCSS Writing Team, April 2011, page 11)

Number and Operation – Fractions[1] 3.NF

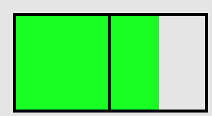
Common Core Cluster

Develop understanding of fractions as numbers.

[1] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.

Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, ½ of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

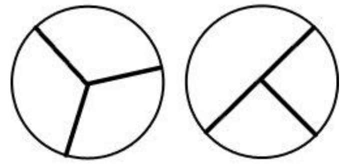
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **partition(ed), equal parts, fraction, equal distance (intervals), equivalent, equivalence, reasonable, denominator, numerator, comparison, compare, ‹, ›, = , justify, inequality**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
3.NF.1 Understand **a fraction** 1/*b* as the quantity formed by 1 part when *a* whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by a parts of size 1/*b*.	This standard refers to the sharing of a whole being partitioned. Fraction models in third grade include only area (parts of a whole) models (circles, rectangles, squares) and number lines. Set models (parts of a group) are not addressed in Third Grade. In 3.NF.1 students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole into equal parts and reasoning about one part of the whole, e.g., if a whole is partitioned into 4 equal parts then each part is ¼ of the whole, and 4 copies of that part make the whole. Next, students build fractions from unit fractions, seeing the numerator 3 of ¾ as saying that ¾ is the quantity you get by putting 3 of the ¼'s together. There is no need to introduce "improper fractions" initially. The importance of specifying the whole  *Without specifying the whole it is not reasonable to ask what fraction is represented by the shaded area. If the left square is the whole, the shaded area represents the fraction $\frac{3}{2}$; if the entire rectangle is the whole, the shaded area represents $\frac{3}{4}$.* (*Progressions for the CCSSM; Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 2)

Some important concepts related to developing understanding of fractions include:

- Understand fractional parts must be equal-sized.

Example Non-example



These are thirds These are NOT thirds

- The number of equal parts tells how many make a whole.
- As the number of equal pieces in the whole increases, the size of the fractional pieces decreases.
- The size of the fractional part is relative to the whole.
    - One-half of a small pizza is relatively smaller than one-half of a large pizza.
- When a whole is cut into equal parts, the denominator represents the number of equal parts.
- The numerator of a fraction is the count of the number of equal parts.
    - ¾ means that there are 3 one-fourths.
    - Students can count *one fourth, two fourths, three fourths.*

Students express fractions as fair sharing or, parts of a whole. They use various contexts (candy bars, fruit, and cakes) and a variety of models (circles, squares, rectangles, fraction bars, and number lines) to develop understanding of fractions and represent fractions. Students need many opportunities to solve word problems that require them to create and reason about fair share.

Initially, students can use an intuitive notion of “same size and same shape” (congruence) to explain why the parts are equal, e.g., when they divide a square into four equal squares or four equal rectangles. Students come to understand a more precise meaning for “equal parts” as “parts with equal measurements.” For example, when a ruler is partitioned into halves or quarters of an inch, they see that each subdivision has the same length. In area models they reason about the area of a shaded region to decide what fraction of the whole it represents.

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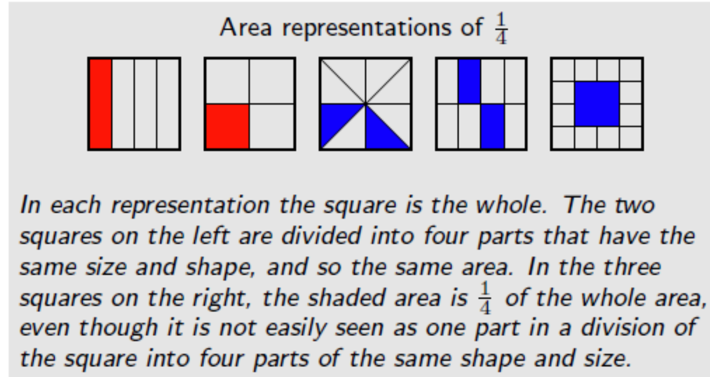

(*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 3)
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3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.

b. Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.
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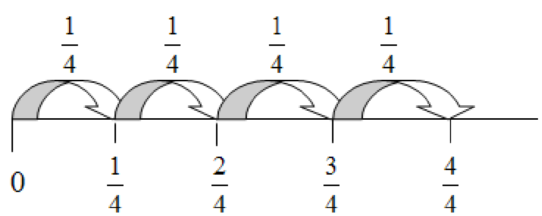
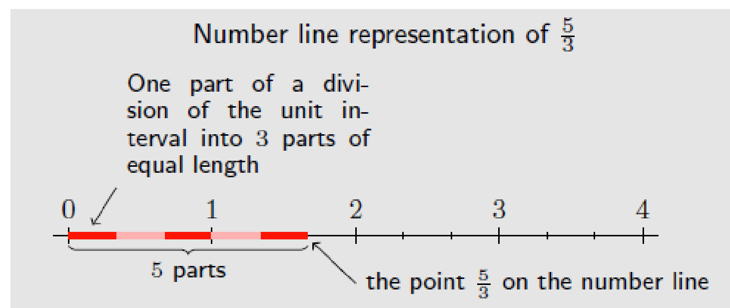
The number line diagram is the first time students work with a number line for numbers that are between whole numbers (e.g., that ½ is between 0 and 1). Students need ample experiences folding linear models (e.g., string, sentence strips) to help them reason about and justify the location of fractions, such that ½ lies exactly halfway between 0 and 1.

In the number line diagram below, the space between 0 and 1 is divided (partitioned) into 4 equal regions. The distance from 0 to the first segment is 1 of the 4 segments from 0 to 1 or ¼ (**3.NF.2a**). Similarly, the distance from 0 to the third segment is 3 segments that are each one-fourth long. Therefore, the distance of 3 segments from 0 is the fraction ¾ (**3.NF.2b**).





(*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 3)
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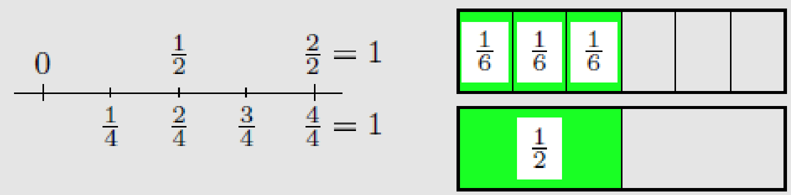
3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **a.** Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. **b.** Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.	An important concept when comparing fractions is to look at the size of the parts and the number of the parts. For example, $\frac{1}{8}$ is smaller than $\frac{1}{2}$ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces. **3.NF.3a** and **3.NF.3b** These standards call for students to use visual fraction models (area models) and number lines to explore the idea of equivalent fractions. Students should only explore equivalent fractions using models, rather than using algorithms or procedures.
c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*	This standard includes writing whole numbers as fractions. The concept relates to fractions as division problems, where the fraction 3/1 is 3 wholes divided into one group. This standard is the building block for later work where students divide a set of objects into a specific number of groups. Students must understand the meaning of a/1.
d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	This standard involves comparing fractions with or without visual fraction models including number lines. Experiences should encourage students to reason about the size of pieces, the fact that 1/3 of a cake is larger than ¼ of the same cake. Since the same cake (the whole) is split into equal pieces, thirds are larger than fourths. In this standard, students should also reason that comparisons are only valid if the wholes are identical. For example, ½ of a large pizza is a different amount than ½ of a small pizza. Students should be given opportunities to discuss and reason about which ½ is larger. Previously, in second grade, students compared lengths using a standard measurement unit. In third grade they build on this idea to compare fractions with the same denominator. They see that for fractions that have the same denominator, the underlying unit fractions are the same size, so the fraction with the greater numerator is greater because it is made of more unit fractions. For example, segment from 0 to ¾ is shorter than the segment from 0 to 5/4 because it measures 3 units of ¼ as opposed to 5 units of ¼, therefore ¾ < 5/4.

Students also see that for unit fractions, the one with the larger denominator is smaller, by reasoning, for example, that in order for more (identical) pieces to make the same whole, the pieces must be smaller. From this they reason that for fractions that have the same numerator, the fraction with the smaller denominator is greater. For example, 2/5 > 2/7, because 1/7 < 1/5, so 2 lengths of 1/7 is less than 2 lengths of 1/5.

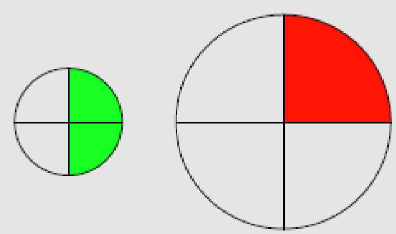
As with equivalence of fractions, it is important in comparing fractions to make sure that each fraction refers to the same whole.

Using the number line and fraction strips to see fraction equivalence



(Progressions for the CCSSM, Number and Operation – Fractions, CCSS Writing Team, August 2011, page 4)

The importance of referring to the same whole when comparing fractions



A student might think that $\frac{1}{4} > \frac{1}{2}$, because a fourth of the pizza on the right is bigger than a half of the pizza on the left.

(*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 4)

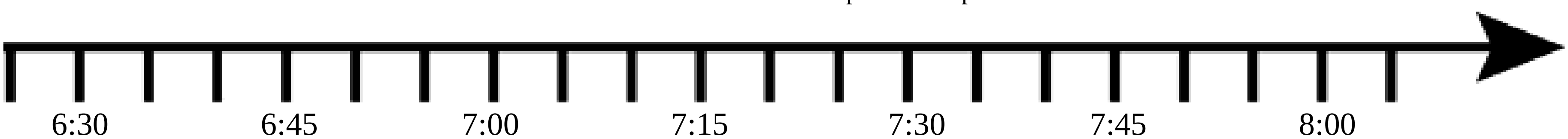
Common Core Cluster

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **estimate, time, time intervals, minute, hour, elapsed time, measure, liquid volume, mass, standard units, metric, gram (g), kilogram (kg), liter (L), milliliter (ML)**

Unpacking Common Core	Unpacking What do these standards mean a child will know and be able to do?
3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	This standard calls for students to solve elapsed time, including word problems. Students could use clock models or number lines to solve. On the number line, students should be given the opportunities to determine the intervals and size of jumps on their number line. Students could use pre-determined number lines (intervals every 5 or 15 minutes) or open number lines (intervals determined by students). In third grade students are expected to tell time within the hour, in fourth grade students are expected to tell time over the hour.

Example:
At 7:00 a.m. Candace wakes up to go to school. It takes her 8 minutes to shower, 9 minutes to get dressed and 17 minutes to eat breakfast. How many minutes does she have until the bus comes at 8:00 a.m.? Use the number line to help solve the problem.



3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[1] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[2] [1] Excludes compound units such as cm3 and finding the geometric volume of a container. [2] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2). (page 89) (Table included at the end of this document for your convenience)	This standard asks for students to reason about the units of mass and volume using units g, kg, and L. Students need multiple opportunities weighing classroom objects and filling containers to help them develop a basic understanding of the size and weight of a liter, a gram, and a kilogram. Milliliters may also be used to show amounts that are less than a liter emphasizing the relationship between smaller units to larger units in the same system. Word problems should only be one-step and include the same units. Students are not expected to do conversions between units, but reason as they estimate, using benchmarks to measure weight and capacity. Example: Students identify 5 things that weigh about one gram. They record their findings with words and pictures. (Students can repeat this for 5 grams and 10 grams.) This activity helps develop gram benchmarks. One large paperclip weighs about one gram. Example: A paper clip weighs about a) a gram, b) 10 grams, c) 100 grams? Explain why. Foundational understandings to help with measure concepts: Understand that larger units can be subdivided into equivalent units (partition). Understand that the same unit can be repeated to determine the measure (iteration). Understand the relationship between the size of a unit and the number of units needed (compensatory principal). Before learning to measure attributes, children need to recognize them, distinguishing them from other attributes. That is, the attribute to be measured has to "stand out" for the student and be discriminated from the undifferentiated sense of amount that young children often have, labeling greater lengths, areas, volumes, and so forth, as "big" or "bigger." These standards do not differentiate between weight and mass. Technically, mass is the amount of matter in an object. Weight is the force exerted on the body by gravity. On the earth's surface, the distinction is not important (on the moon, an object would have the same mass, would weigh less due to the lower gravity). *(Progressions for the CCSSM*, *Geometric Measurement*, CCSS Writing Team, June 2012, page 2)

Much of the work involving measure supports the work that is emphasized in third on multiplication.
Example:

Table 1: Multiplication and division situations for measurement

	Unknown Product $A \times B = \square$	Group Size Unknown $A \times \square = C$ and $C \div A = \square$	Number of Groups Unknown $\square \times B = C$ and $C \div B = \square$
Grouped Objects (Units of Units)	You need A lengths of string, each B inches long. How much string will you need altogether?	You have C inches of string, which you will cut into A equal pieces. How long will each piece of string be?	You have C inches of string, which you will cut into pieces that are B inches long. How many pieces of string will you have?
Arrays of Objects (Spatial Structuring)	What is the area of a A cm by B cm rectangle?	A rectangle has area C square centimeters. If one side is A cm long, how long is a side next to it?	A rectangle has area C square centimeters. If one side is B cm long, how long is a side next to it?
Compare	A rubber band is B cm long. How long will the rubber band be when it is stretched to be A times as long?	A rubber band is stretched to be C cm long and that is A times as long as it was at first. How long was the rubber band at first?	A rubber band was B cm long at first. Now it is stretched to be C cm long. How many times as long is the rubber band now as it was at first?

Adapted from box 2-4 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33. Note that Grade 3 work does not include Compare problems with “times as much,” see the Operations and Algebraic Thinking Progression, Table 3, also p. 29.

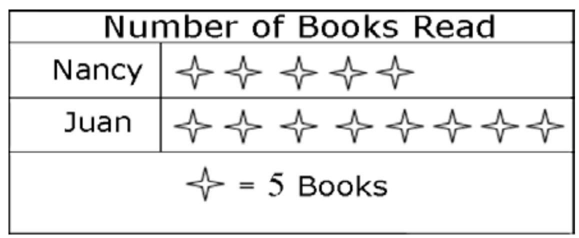
(Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, June 2012, page 19)

Common Core Cluster

Represent and interpret data.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **scale, scaled picture graph, scaled bar graph, line plot, data**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*	Students should have opportunities reading and solving problems using scaled graphs before being asked to draw one. Work with scaled graphs builds on students' understanding of multiplication and division. The following graphs provided below all use five as the scale interval, but students should experience different intervals to further develop their understanding of scale graphs and number facts. While exploring data concepts, students should Pose a question, Collect data, Analyze data, and Interpret data (PCAI). Students should be graphing data that is relevant to their lives Example: Pose a question: Student should come up with a question. What is the typical genre read in our class? Collect and organize data: student survey Pictographs: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, categories, category label, key, and data. How many more books did Juan read than Nancy?



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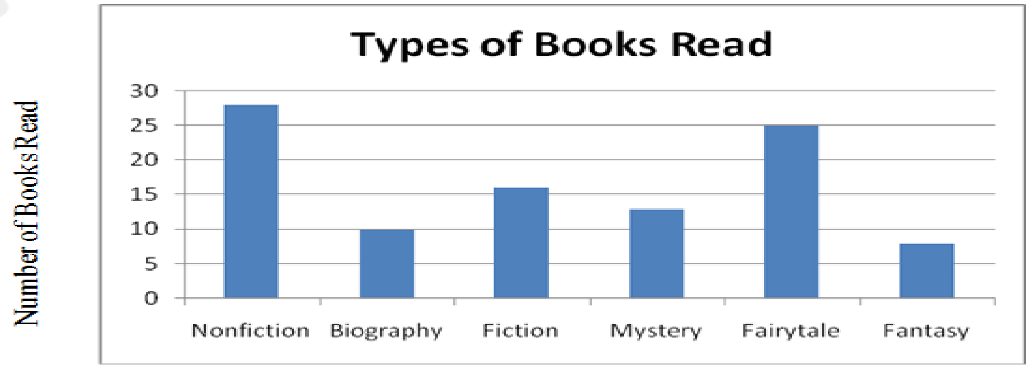
Single Bar Graphs: Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.


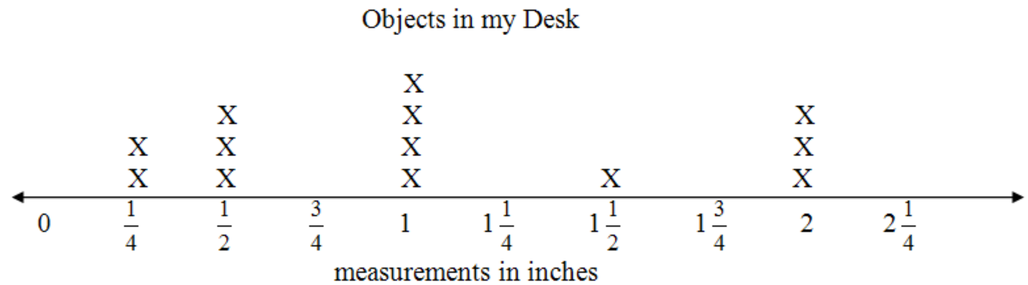
Analyze and Interpret data:

How many more nonfiction books were read than fantasy books?
Did more people read biography and mystery books or fiction and fantasy books?
About how many books in all genres were read?
Using the data from the graphs, what type of book was read more often than a mystery but less often than a fairytale?
What interval was used for this scale?
What can we say about types of books read? What is a typical type of book read? (beyond standard)
If you were to purchase a book for the class library which would be the best genre? Why? (beyond standard)

</td>
</tr>
<tr>
<td>3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.</td>
<td>Students in second grade measured length in whole units using both metric and U.S. customary systems. It’s important to review with students how to read and use a standard ruler including details about halves and quarter marks on the ruler. Students should connect their understanding of fractions to measuring to one-half and one-quarter inch. Third graders need many opportunities measuring the length of various objects in their environment.

This standard provides a context for students to work with fractions by measuring objects to a quarter of an inch.</td>
</tr>
</table>

Example:
Measure objects in your desk to the nearest ½ or ¼ of an inch, display data collected on a line plot. How many objects measured ¼? ½? etc…



In Grade 3, students are beginning to learn fraction concepts (3.NF). They understand fraction equivalence in simple cases, and they use visual fraction models to represent and order fractions. Grade 3 students also measure lengths using rulers marked with halves and fourths of an inch. They use their developing knowledge of fractions and number lines to extend their work from the previous grade by working with measurement data involving fractional measurement values.

For example, every student in the class might measure the height of a bamboo shoot growing in the classroom, leading to the data set shown in the table. (Illustration below shows a larger data set than students would normally work with in elementary grades.)

To make a line plot from the data in the table, the student can determine the greatest and least values in the data: 13 ½ inches and 14 ¾ inches. The student can draw a segment of a number line diagram that includes these extremes, with tick marks indicating specific values on the measurement scale. This is just like part of the scale on a ruler. Having drawn the number line diagram, the student can proceed through the data set recording each observation by drawing a symbol, such as a dot, above the proper tick mark. As with Grade 2 line plots, if a particular data value appears many times in the data set, dots will "pile up" above that value. There is no need to sort the observations, or to do any counting of them, before producing the line plot. Students can pose questions about data presented in line plots, such as how many students obtained measurements larger than 14 ¼ inches.

Students' measurements of a statue and of a bamboo shoot

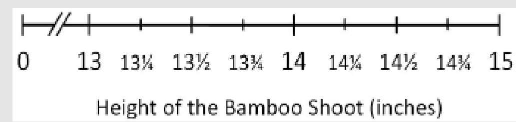
Statue measurements

Student's initials	Student's measured value (inches)
W.B.	64
D.W.	65
H.D.	65
G.W.	65
V.Y.	67
T.T.	66
D.F.	67
B.H.	65
H.H.	63
V.H.	64
I.O.	64
W.N.	65
B.P.	69
V.A.	65
H.L.	66
O.M.	64
L.E.	65
M.J.	66
T.D.	66
K.P.	64
H.N.	65
W.M.	67
C.Z.	64
J.I.	66
M.S.	66
T.C.	65
G.V.	67
O.F.	65

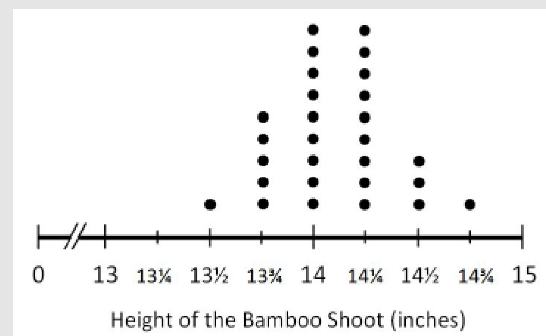
Bamboo shoot measurements

Student's initials	Height value (inches)
W.B.	13 3/4
D.W.	14 1/2
H.D.	14 1/4
G.W.	14 3/4
V.Y.	14 1/4
T.T.	14 1/2
D.F.	14
B.H.	13 1/2
H.H.	14 1/4
V.H.	14 1/4
I.O.	14 1/4
W.N.	14
B.P.	14 1/2
V.A.	13 3/4
H.L.	14
O.M.	13 3/4
L.E.	14 1/4
M.J.	13 3/4
T.D.	14 1/4
K.P.	14
H.N.	14
W.M.	14
C.Z.	13 3/4
J.I.	14
M.S.	14 1/4
T.C.	14
G.V.	14
O.F.	14 1/4

A scale for a line plot of the bamboo shoot data



A line plot of the bamboo shoot data



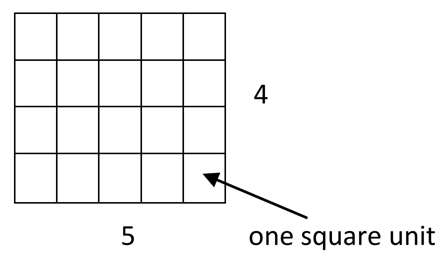
(*Progressions for the CCSSM, Measurement Data*, CCSS Writing Team, June 2011, page 10)

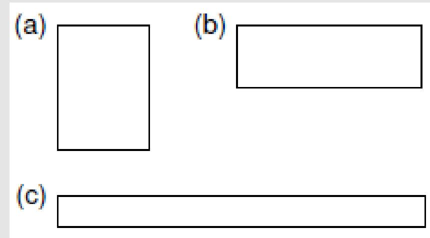
Common Core Cluster

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

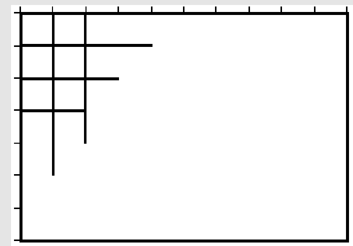
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **attribute, area, square unit, plane figure, gap, overlap, square cm, square m , square in., square ft, nonstandard units, tiling, side length, decomposing**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	These standards call for students to explore the concept of covering a region with "unit squares," which could include square tiles or shading on grid or graph paper. Based on students' development, they should have ample experiences filling a region with square tiles before transitioning to pictorial representations on graph paper. 

	Example: **Which rectangle covers the most area?**  These rectangles are formed from unit squares (tiles students have used) although students are not informed of this or the rectangle's dimensions: (a) 4 by 3, (b) 2 by 6, and (c) 1 row of 12. Activity from Lehrer, et al., 1998, "Developing understanding of geometry and space in the primary grades," in R. Lehrer & D. Chazan (Eds.), *Designing Learning Environments for Developing Understanding of Geometry and Space*, Lawrence Erlbaum Associates. *(Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 16)
3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	Students should be counting the square units to find the area could be done in metric, customary, or non-standard square units. Using different sized graph paper, students can explore the areas measured in square centimeters and square inches. The task shown above would provides a great experience for students to tile a region and count the number of square units
3.MD.7 Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.	Students can learn how to multiply length measurements to find the area of a rectangular region. But, in order that they make sense of these quantities, they must first learn to interpret measurement of rectangular regions as a multiplicative relationship of the number of square units in a row and the number of rows. This relies on the development of spatial structuring. To build from spatial structuring to understanding the number of area-units as the product of number of units in a row and number of rows, students might draw rectangular arrays of squares and learn to determine the number of squares in each row with increasingly sophisticated strategies, such as skip-counting the number in each row and eventually multiplying the number in each row by the number of rows. They learn to partition a rectangle into identical squares by anticipating the final structure and forming the array by drawing line segments to form rows and columns. They use skip counting and multiplication to determine the number of squares in the array.

Many activities that involve seeing and making arrays of squares to form a rectangle might be needed to build robust conceptions of a rectangular area structured into squares.

Incomplete array



To determine the area of this rectangular region, students might be encouraged to construct a row, corresponding to the indicated positions, then repeating that row to fill the region. Cutouts of strips of rows can help the needed spatial structuring and reduce the time needed to show a rectangle as rows or columns of squares. Drawing all of the squares can also be helpful, but it is slow for larger rectangles. Drawing the unit lengths on the opposite sides can help students see that joining opposite unit end-points will create the needed unit square grid.

Students should understand and explain why multiplying the side lengths of a rectangle yields the same measurement of area as counting the number of tiles (with the same unit length) that fill the rectangle's interior For example, students might explain that one length tells how many unit squares in a row and the other length tells how many rows there are.
(Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, June 2012, page 17)

Students should tile rectangle then multiply the side lengths to show it is the same.

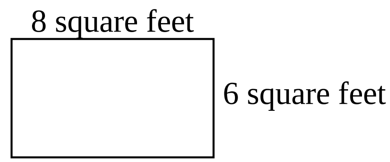
To find the area one could count the squares or multiply 3 x 4 = 12.

1	2	3	4
5	6	7	8
9	10	11	12

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

Students should solve real world and mathematical problems

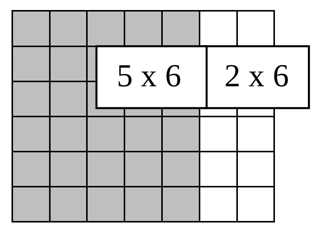
Example:
Drew wants to tile the bathroom floor using 1 foot tiles. How many square foot tiles will he need?



Students might solve problems such as finding all the rectangular regions with whole-number side lengths that have an area of 12 area-units, doing this for larger rectangles (e.g., enclosing 24, 48, 72 area-units), making sketches rather than drawing each square. Students learn to justify their belief they have found all possible solutions. *(Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 18)

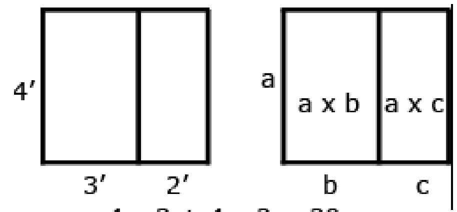
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning.

This standard extends students' work with the distributive property. For example, in the picture below the area of a 7 x 6 figure can be determined by finding the area of a 5 x 6 and 2 x 6 and adding the two sums.



Using concrete objects or drawings students build competence with composition and decomposition of shapes, spatial structuring, and addition of area measurements, students learn to investigate arithmetic properties using area models. For example, they learn to rotate rectangular arrays physically and mentally, understanding that their areas are preserved under rotation, and thus, for example, 4 x 7 = 7 x 4, illustrating the commutative property of multiplication. Students also learn to understand and explain that the area of a rectangular region of, for example, 12 length-units by 5 length-units can be found either by multiplying 12 x 5, or by adding two products, e.g., 10 x 5 and 2 x 5, illustrating the distributive property. *(Progressions for the CCSSM, Geometric Measurement,* CCSS Writing Team, June 2012, page 18)

Example:



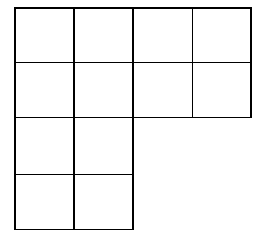
4 x 3 + 4 x 2 = 20

4 (3 + 2) = 20

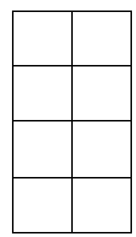
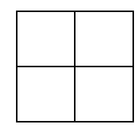
4 x 5 = 20

d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

This standard uses the word rectilinear. A rectilinear figure is a polygon that has all right angles.



How could this figure be decomposed to help find the area?

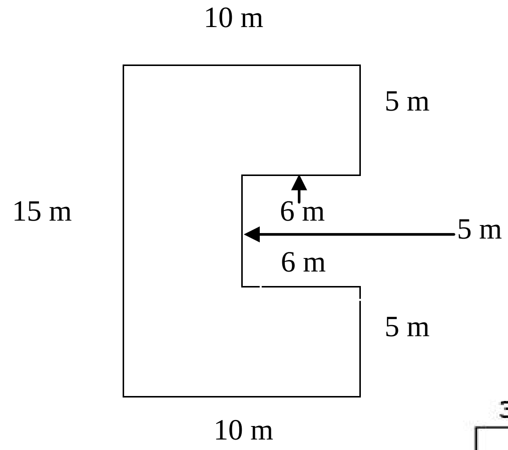
 This portion of the decomposed figure is a 4 x 2.	 This portion of the decomposed figure is 2 x 2.

4 x 2 = 8 and 2 x 2 = 4

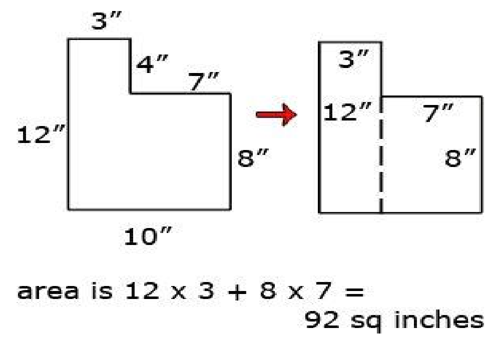
So 8 + 4 = 12

Therefore the total area of this figure is 12 square units

Example:
A storage shed is pictured below. What is the total area?
How could the figure be decomposed to help find the area?



Example:
Students can decompose a rectilinear figure into different rectangles. They find the area of the figure by adding the areas of each of the rectangles together.



With strong and distinct concepts of both perimeter and area established, students can work on problems to differentiate their measures. For example, they can find and sketch rectangles with the same perimeter and different areas or with the same area and different perimeters and justify their claims Differentiating perimeter from area is facilitated by having students draw congruent rectangles and measure, mark off, and label the unit lengths all around the perimeter on one rectangle, then do the same on the other rectangle but also draw the square units. This enables students to see the units involved in length and area and find patterns in finding the lengths and areas of non-square and square rectangles. Students can continue to describe and show the units involved in perimeter and area after they no longer need these. *(Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 18)

Common Core Cluster

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

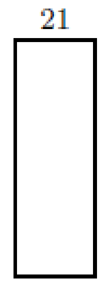
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **attribute, perimeter, plane figure, linear, area, polygon, side length**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	Students develop an understanding of the concept of perimeter through various experiences, such as walking around the perimeter of a room, using rubber bands to represent the perimeter of a plane figure on a geoboard, or tracing around a shape on an interactive whiteboard. They find the perimeter of objects; use addition to find perimeters; and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles. Students should also strategically use tools, such as geoboards, tiles, and graph paper to find all the possible rectangles that have a given perimeter (e.g., find the rectangles with a perimeter of 14 cm.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Following this experience, students can reason about connections between their representations, side lengths, and the perimeter of the rectangles. Given a perimeter and a length or width, students use objects or pictures to find the missing length or width. They justify and communicate their solutions using words, diagrams, pictures, numbers, and an interactive whiteboard. Students use geoboards, tiles, graph paper, or technology to find all the possible rectangles with a given area (e.g. find the rectangles that have an area of 12 square units.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Students then investigate the perimeter of the rectangles with an area of 12.

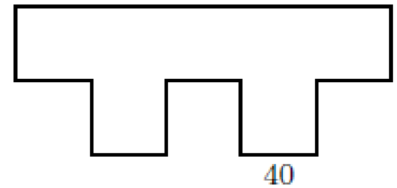
Area	Length	Width	Perimeter
12 sq. in.	1 in.	12 in.	26 in.
12 sq. in.	2 in.	6 in.	16 in.
12 sq. in	3 in.	4 in.	14 in.
12 sq. in	4 in.	3 in.	14 in.
12 sq. in	6 in.	2 in.	16 in.
12 sq. in	12 in.	1 in.	26 in.

	The patterns in the chart allow the students to identify the factors of 12, connect the results to the commutative property, and discuss the differences in perimeter within the same area. This chart can also be used to investigate rectangles with the same perimeter. It is important to include squares in the investigation. A perimeter is the boundary of a two-dimensional shape. For a polygon, the length of the perimeter is the sum of the lengths of the sides. Initially, it is useful to have sides marked with unit length marks, allowing students to count the unit lengths. Later, the lengths of the sides can be labeled with numerals. As with all length tasks, students need to count the length-units and not the end-points. Next, students learn to mark off unit lengths with a ruler and label the length of each side of the polygon. For rectangles, parallelograms, and regular polygons, students can discuss and justify faster ways to find the perimeter length than just adding all of the lengths. Rectangles and parallelograms have opposite sides of equal length, so students can double the lengths of adjacent sides and add those numbers or add lengths of two adjacent sides and double that number. A regular polygon has all sides of equal length, so its perimeter length is the product of one side length and the number of sides. Perimeter problems for rectangles and parallelograms often give only the lengths of two adjacent sides or only show numbers for these sides in a drawing of the shape. The common error is to add just those two numbers. Having students first label the lengths of the other two sides as a reminder is helpful. Students then find unknown side lengths in more difficult “missing measurements” problems and other types of perimeter problems. *(Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 16)

Missing measurements and other perimeter problems



The perimeter of this rectangle is 168 length units. What are the lengths of the three unlabeled sides?



Assume all short segments are the same length and all angles are right

Compare these problems with the "missing measurements" problems of Grade 2.

Another type of perimeter problem is to draw a robot on squared grid paper that meets specific criteria. All the robot's body parts must be rectangles. The perimeter of the head might be 36 length-units, the body, 72; each arm, 24; and each leg, 72. Students are asked to provide a convincing argument that their robots meet these criteria (MP3). Next, students are asked to figure out the area of each of their body parts (in square units). These are discussed, with students led to reflect on the different areas that may be produced with rectangles of the same perimeter. These types of problems can be also presented as turtle geometry problems. Students create the commands on paper and then give their commands to the Logo turtle to check their calculations. For turtle length units, the perimeter of the head might be 300 length-units, the body, 600; each arm, 400; and each leg, 640.

(*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 16)

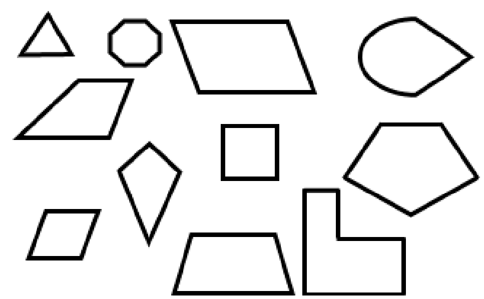
Geometry 3.G

Common Core Cluster

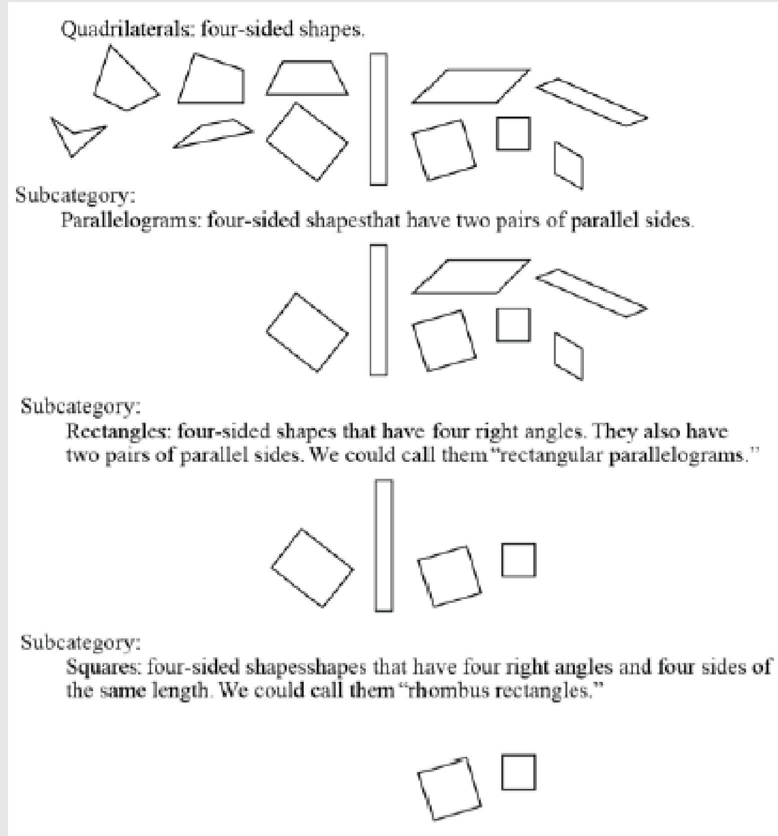
Reason with shapes and their attributes.
Students describe, analyze, and compare properties of two dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are**: properties[1], attributes[1], features[1], , quadrilateral, open figure, closed figure , three-sided, 2-dimensional, rhombi, rectangles, and squares are subcategories of quadrilaterals, polygon, rhombus/rhombi, rectangle, square, partition, unit fraction, kite, parallelogram, examples, and non-examples**
From previous grades: **triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, cone, cylinder, sphere, sides, vertices, corners**

[1]The term "**property**" in these standards is reserved for those attributes that indicate a relationship between components of shapes. Thus, "having parallel sides" or "having all sides of equal lengths" are properties. "**Attributes**" and "**features**" are used interchangeably to indicate any characteristic of a shape, including properties, and other defining characteristics (e.g., straight sides) and non-defining characteristics (e.g., "right-side up").
(Progressions for the CCSSM, *Geometry*, CCSS Writing Team, June 2012, page 3 footnote)

Common Core Standard	**Unpacking** What do these standards mean a child will know and be able to do?
3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	In second grade, students identify and draw triangles, quadrilaterals, pentagons, and hexagons. Third graders build on this experience and further investigate quadrilaterals (technology may be used during this exploration). Students recognize shapes that are and are not quadrilaterals by examining the properties of the geometric figures. They conceptualize that a quadrilateral must be a closed figure with four straight sides and begin to notice characteristics of the angles and the relationship between opposite sides. Students should be encouraged to provide details and use proper vocabulary when describing the properties of quadrilaterals. They sort geometric figures (see examples below) and identify squares, rectangles, and rhombuses as quadrilaterals.  Third grade students have built a firm foundation of several shape categories, these categories can be the raw material for thinking about the relationships between classes. Students should classify shapes by attributes and drawing shapes that fit specific categories. Example: students can form larger categories, such as the class of all shapes with four sides, or quadrilaterals, and recognize that it includes other categories, such as squares, rectangles, rhombuses, parallelograms, and trapezoids. They also recognize that there are quadrilaterals that are not in any of those subcategories.

Quadrilaterals and some special kinds of quadrilaterals



1

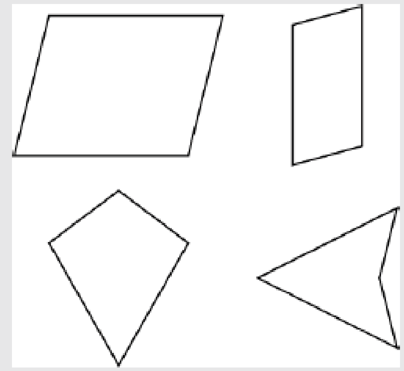
The representations above might be used by teachers in class. Note that the left-most four shapes in the first section at the top left have four sides but do not have properties that would place them in any of the other categories shown (parallelograms, rectangles, squares).

The standards do **not** require the above representation be constructed by students, but students should be able to draw examples of quadrilaterals that are not in the subcategories.

(Progressions for the CCSSM, Geometry, CCSS Writing Team, June 2012, page 13)

Example:

Quadrilaterals that are not rectangles



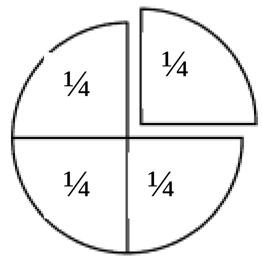
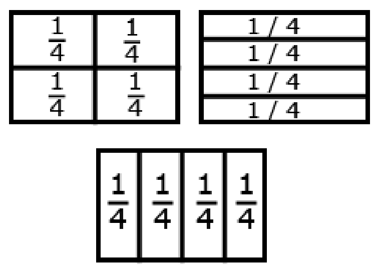
These quadrilaterals have two pairs of sides of the same length but are not rectangles. A kite is on lower left and a deltoid is at lower right.

Parallelograms include: squares, rectangles, rhombi, or other shapes that have two pairs of parallel sides. Also, the broad category quadrilaterals include all types of parallelograms, trapezoids and other four-sided figures.
Example:
Draw a picture of a quadrilateral. Draw a picture of a rhombus.
How are they alike? How are they different?
Is a quadrilateral a rhombus? Is a rhombus a quadrilateral? Justify your thinking.

A **kite** is a quadrilateral whose four sides can be grouped into two pairs of equal-length sides that are beside each other.

The notion of congruence ("same size and same shape") may be part of classroom conversation but the concepts of congruence and similarity do **not** appear until middle school.

<u>TEACHER NOTE:</u> In the U.S., the term "trapezoid" may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with *at least* one pair of parallel sides. The exclusive definition states: **A trapezoid is a quadrilateral with *exactly* one pair of parallel sides.** With this definition, a parallelogram is not a trapezoid. North Carolina has adopted the exclusive definition. (*Progressions for the CCSSM: Geometry*, The Common Core Standards Writing Team, June 2012.)

3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*	In third grade students start to develop the idea of a fraction more formally, building on the idea of partitioning a whole into equal parts. The whole can be a shape such as a circle or rectangle. In Grade 4, this is extended to include wholes that are collections of objects. This standard also builds on students' work with fractions and area. Students are responsible for partitioning shapes into halves, thirds, fourths, sixths and eighths. Example: This figure was partitioned/divided into four equal parts. Each part is ¼ of the total area of the figure.  Given a shape, students partition it into equal parts, recognizing that these parts all have the same area. They identify the fractional name of each part and are able to partition a shape into parts with equal areas in several different ways. 

Some examples used in this document are from the Arizona Mathematics Education Department

Glossary

Table 1 Common addition and subtraction situations[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**[2]
Put Together/ Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[2]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[3]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[4]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2 Common multiplication and division situations[1]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.
[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3 The properties of operations

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

REFERENCES

Common Core Standards Writing Team (Bill McCullum, lead author). *Progressions for the common core state standards in mathematics: Geometry* (draft). June 23, 2012. Retrieved from: www.commoncoretools.wordpress.com.

Common Core Standards Writing Team (Bill McCullum, lead author). *Progressions for the common core state standards in mathematics: Geometric measurement* (draft). June 23, 2012. Retrieved from: www.commoncoretools.wordpress.com.

Common Core Standards Writing Team (Bill McCullum, lead author). *Progressions for the common core state standards in mathematics: K-3, Categorical data; Grades 2-5, Measurement Data* (draft). June 20, 2011. Retrieved from: www.commoncoretools.wordpress.com.

Common Core Standards Writing Team (Bill McCullum, lead author). *Progressions for the common core state standards in mathematics: K, Counting and cardinality; K-5, operations and algebraic thinking* (draft). May 29, 2011. Retrieved from: www.commoncoretools.wordpress.com.

Common Core Standards Writing Team (Bill McCullum, lead author). *Progressions for the common core state standards in mathematics: K-5, Number and operations in base ten* (draft). April 7, 2011. Retrieved from: www.commoncoretools.wordpress.com.

Common Core Standards Writing Team (Bill McCullum, lead author). *Progressions for the common core state standards in mathematics: 3-5 Number and operations - fractions* (draft). July 12, 2011. Retrieved from: www.commoncoretools.wordpress.com.

Copley, J. (2010). *The young child and mathematics*. Washington DC: NAEYC.

Fosnot, C. & Dolk, M. (2001). *Young mathematicians at work: Constructing number sense, addition, and subtraction*. Portsmouth: Heinemann.

Fosnot, C. & Dolk, M. (2001). *Young mathematicians at work: Constructing multiplication and division*. Portsmouth: Heinemann.

Fosnot, C. & Dolk, M. (2001). *Young mathematicians at work: Constructing fractions, decimals, and percents*. Portsmouth: Heinemann.

Chapin, S. & Johnson, A. (2006). *Math matters: Grade K-8 understanding the math you teach.* Sausalito: Math Solution Publications.

Van de Walle, J., Lovin, L. (2006). *Teaching student-centered mathematics 3-5*. Boston: Pearson.

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