North Carolina Department of Public Instruction
INSTRUCTIONAL SUPPORT TOOLS
FOR ACHIEVING NEW STANDARDS

4th *Grade Mathematics* ● Unpacked Content

This document is designed to help North Carolina educators teach the Common Core (Standard Course of Study). NCDPI staff are continually updating and improving these tools to better serve teachers. This document was written by the NCDPI Mathematics Consultants with the collaboration of many educators from across the state.

What is the purpose of this document?
To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the CCSS.

What is in the document?
Descriptions of what each standard means a student will know, understand and be able to do. The "unpacking" of the standards done in this document is an effort to answer a simple question "What does this standard mean that a student must know and be able to do?" and to ensure the description is helpful, specific and comprehensive for educators.

How do I send Feedback?
We intend the explanations and examples in this document to be helpful and specific. That said, we believe that as this document is used, teachers and educators will find ways in which the unpacking can be improved and made ever more useful. Please send feedback to us at kitty.rutherford@dpi.nc.gov or denise.schulz@dpi.nc.gov and we will use your input to refine our unpacking of the standards. Thank You!

Just want the standards alone?
You can find the standards alone at http://corestandards.org/the-standards

Standards for Mathematical Practices

The Common Core State Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

Mathematic Practices	Explanations and Examples
1. Make sense of problems and persevere in solving them.	Mathematically proficient students in grade 4 know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
2. Reason abstractly and quantitatively.	Mathematically proficient fourth grade students should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others.	In fourth grade mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	Mathematically proficient fourth grade students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
5. Use appropriate tools strategically.	Mathematically proficient fourth grader students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.
6. Attend to precision.	As fourth grader students develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
7. Look for and make use of structure.	In fourth grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.
8. Look for and express regularity in repeated reasoning.	Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Grade 4 Critical Areas

The Critical Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction. The Critical Areas for fourth grade can be found on page 27 in the *Common Core State Standards for Mathematics*.

1. **Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends.**

Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

2. **Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.**

Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

3. **Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.**

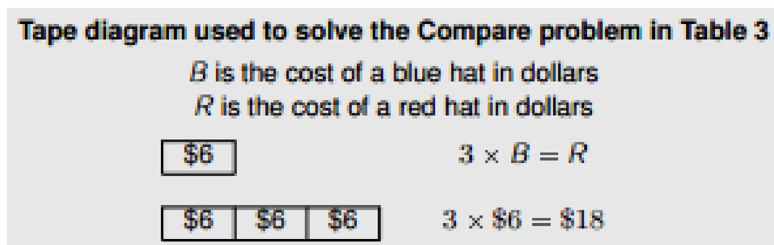
Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

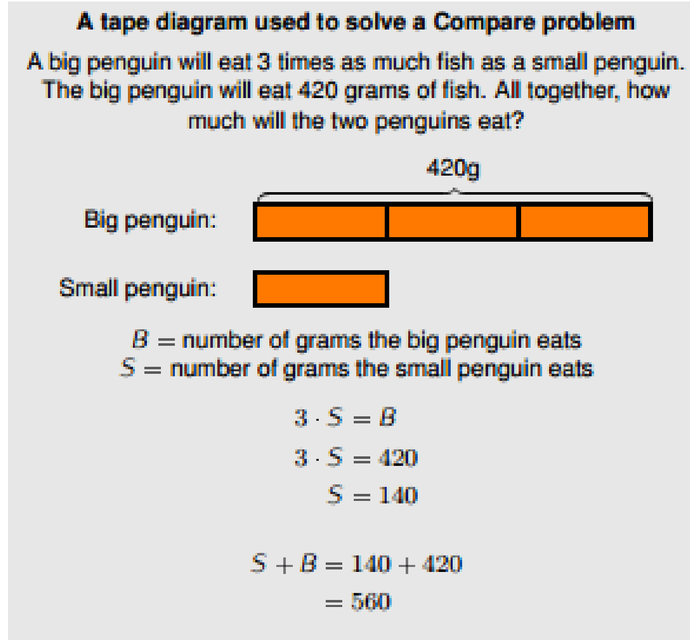
Operations and Algebraic Thinking 4.OA

Common Core Cluster

Use the four operations with whole numbers to solve problems.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multiplication/multiply, division/divide, addition/add, subtraction/subtract, equations, unknown, remainders, reasonableness, mental computation, estimation, rounding**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	A *multiplicative comparison* is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., "*a* is *n* times as much as *b*"). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times. Students should be given opportunities to write and identify equations and statements for multiplicative comparisons. Example: *5 x 8 = 40.* Sally is five years old. Her mom is eight times older. How old is Sally's Mom? *5 x 5 = 25* Sally has five times as many pencils as Mary. If Mary has 5 pencils, how many does Sally have?
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1] [1] See Glossary, Table 2. (page 89) (Table included at the end of this document for your convenience)	This standard calls for students to translate comparative situations into equations with an unknown and solve. Students need many opportunities to solve contextual problems. Refer to Glossary, Table 2(page 89) For more examples (table included at the end of this document for your convenience) In an additive comparison, the underling question is *what amount would be added to one quantity* in order to result in the other. In a multiplicative comparison, the underlying question is *what factor would multiply one quantity* in order to result in the other. 



(*Progressions for the CCSSM; Operations and Algebraic Thinking* , CCSS Writing Team, May 2011, page 29)

Examples:
Unknown Product: A blue scarf costs $3. A red scarf costs 6 times as much. How much does the red scarf cost? (3 x 6 = p).
Group Size Unknown: A book costs $18. That is 3 times more than a DVD. How much does a DVD cost? (18 ÷ p = 3 or 3 x p = 18).
Number of Groups Unknown: A red scarf costs $18. A blue scarf costs $6. How many times as much does the red scarf cost compared to the blue scarf? (18 ÷ 6 = p or 6 x p = 18).

When distinguishing multiplicative comparison from additive comparison, students should note that

- additive comparisons focus on the difference between two quantities (e.g., Deb has 3 apples and Karen has 5 apples. How many more apples does Karen have?). A simple way to remember this is, “How many more?”
- multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other (e.g., Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run?). A simple way to remember this is “How many times as much?” or “How many times as many?”

<table>
<tr>
<td>4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</td>
<td>
The focus in this standard is to have students use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems using all four operations.
Example:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total?
Some typical estimation strategies for this problem:
<table>
<tr>
<td>Student 1
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.</td>
<td>Student 2
I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.</td>
<td>Student 3
I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530.</td>
</tr>
</table>
The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer.
Examples continued on the next page.
Example 2:
Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?
<table>
<tr>
<td>Student 1
First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I’m trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.</td>
<td>Student 2
First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40+20=60. 300-60 = 240, so we need about 240 more bottles.</td>
</tr>
</table>
</td>
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This standard references interpreting remainders. Remainders should be put into context for interpretation. ways to address remainders:

- Remain as a left over
- Partitioned into fractions or decimals
- Discarded leaving only the whole number answer
- Increase the whole number answer up one
- Round to the nearest whole number for an approximate result

Example:
Write different word problems involving **44 ÷ 6 = ?** where the answers are best represented as:

Problem A: 7
Problem B: 7 r 2
Problem C: 8
Problem D: 7 or 8
Problem E: $7\frac{2}{6}$

possible solutions:

Problem **A: 7.** Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill? 44 ÷ *6 = p; p = 7 r 2. Mary can fill 7 pouches completely.*

Problem **B: 7 r 2.** Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches could she fill and how many pencils would she have left? 44 ÷ *6 = p; p = 7 r 2; Mary can fill 7 pouches and have 2 left over.*

Problem **C: 8.** Mary had 44 pencils. Six pencils fit into each of her pencil pouches. What would the fewest number of pouches she would need in order to hold all of her pencils? 44 ÷ *6 = p; p = 7 r 2; Mary can needs 8 pouches to hold all of the pencils.*

Problem **D: 7 or 8.** Mary had 44 pencils. She divided them equally among her friends before giving one of the leftovers to each of her friends. How many pencils could her friends have received? 44 ÷ *6 = p; p = 7 r 2; Some of her friends received 7 pencils. Two friends received 8 pencils.*

Problem **E: 7** $\frac{2}{6}$**.** Mary had 44 pencils and put six pencils in each pouch. What fraction represents the number of pouches that Mary filled? 44 ÷ *6 = p; p =* **7** $\frac{2}{6}$

Example:
There are 1,128 students going on a field trip. If each bus held 30 students, how many buses are needed? *(1,128 ÷ 30 = b; b = 37 R 6; They will need 38 buses because 37 busses would not hold all of the students).*

Students need to realize in problems, such as the example above, that an extra bus is needed for the 8 students that are left over.

	Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to: • front-end estimation with adjusting (using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts), • clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate), • rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values), • using friendly or compatible numbers such as factors (students seek to fit numbers together - e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000), • using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate).

Common Core Cluster

Gain familiarity with factors and multiples.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multiplication/multiply, division/divide, factor pairs, factor, multiple, prime, composite**

Common Core Standard	**Unpacking** What do these standards mean a child will know and be able to do?
4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8. A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number. Prime vs. Composite: A prime number is a number greater than 1 that has only 2 factors, 1 and itself. Composite numbers have more than 2 factors. Students investigate whether numbers are prime or composite by

- building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into only 2 rectangles, 1 x 7 and 7 x 1, therefore it is a prime number)
- finding factors of the number

Students should understand the process of finding factor pairs so they can do this for any number 1 - 100,
Example:
Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.

Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20).

Example:
Factors of 24: 1, 2, 3, 4, 6, 8,12, 24
Multiples: 1, 2, 3, 4, 5…24
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24
3, 6, 9, 12, 15, 18, 21, 24
4, 8, 12, 16, 20, 24
8, 16, 24
12, 24
24

To determine if a number between1-100 is a multiple of a given one-digit number, some helpful hints include the following:

- all even numbers are multiples of 2
- all even numbers that can be halved twice (with a whole number result) are multiples of 4
- all numbers ending in 0 or 5 are multiples of 5

Common Core Cluster

Generate and analyze patterns.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **pattern (number or shape), pattern rule**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations. Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features. Example: (see table below) After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule. Example: Rule: Starting at 1, create a pattern that starts at 1 and multiplies each number by 3. Stop when you have 6 numbers. Students write 1, 3, 9, 27, 81, 243. Students notice that all the numbers are odd and that the sums of the digits of the 2 digit numbers are each 9. Some students might investigate this beyond 6 numbers. Another feature to investigate is the patterns in the differences of the numbers (3 - 1 = 2, 9 - 3 = 6, 27 - 9 = 18, etc.)

Example:

Pattern	Rule	Feature(s)
3, 8, 13, 18, 23, 28, …	Start with 3, add 5	The numbers alternately end with a 3 or 8
5, 10, 15, 20 …	Start with 5, add 5	The numbers are multiples of 5 and end with either 0 or 5. The numbers that end with 5 are products of 5 and an odd number. The numbers that end in 0 are products of 5 and an even number.

This standard calls for students to describe features of an arithmetic number pattern or shape pattern by identifying the rule, and features that are not explicit in the rule. A t-chart is a tool to help students see number patterns.

Example:
There are 4 beans in the jar. Each day 3 beans are added. How many beans are in the jar for each of the first 5 days?

Day	**Operation**	**Beans**
0	3 x 0 + 4	4
1	3 x 1 + 4	7
2	3 x 2 + 4	10
3	3 x 3 + 4	13
4	3 x 4 + 4	16
5	3 x 5 + 4	19

This standard begins with a small focus on reasoning about a number or shape pattern, connecting a rule for a given pattern with its sequence of numbers or shapes. Patterns that consist of repeated sequences of shapes or growing sequences of designs can be appropriate for the grade. For example, students could examine a sequence of dot designs in which each design has 4 more dots than the previous one and they could reason about how the dots are organized in the design to determine the total number of dots in the 100^{th} design. In examining numerical sequences, fourth graders can explore rules of repeatedly adding the same whole number or repeatedly multiplying by the same whole number. Properties of repeating patterns of shapes can be explored with division. For example, to determine the 100^{th} shape in a pattern that consists of repetitions of the sequence "square, circle, triangle," the fact that when we divide 100 by 3 the whole number quotient is 33 with remainder 1 tells us that after 33 full repeats, the 99th shape will be a triangle (the last shape in the repeating pattern), so the 100th shape is the first shape in the pattern, which is a square. Notice that the Standards do not require students to infer or guess the underlying rule for a pattern, but rather ask them to generate a pattern from a given rule and identify features of the given pattern. (*Progressions for the CCSSM; Operations and Algebraic Thinking* , CCSS Writing Team, May 2011, page 31)

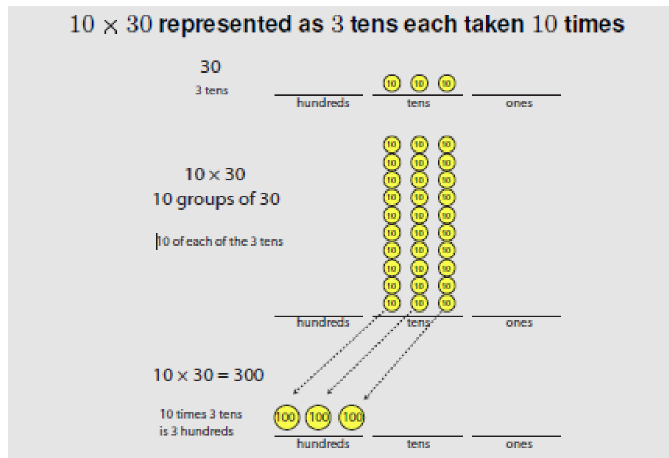
Number and Operation in Base Ten[1] 4.NBT

Common Core Standard and Cluster

Generalize place value understanding for multi-digit whole numbers.

[1]Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, greater than, less than, equal to, ‹, ›, =, comparisons/compare, round, base-ten numerals (standard from), number name (written form), expanded form, inequality, expression**

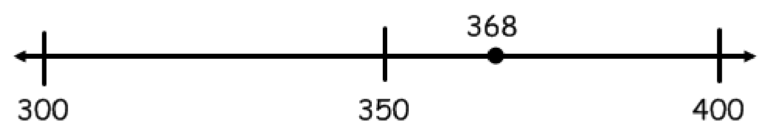
	Unpacking What do these standards mean a child will know and be able to do?
4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*	This standard calls for students to extend their understanding of place value related to multiplying and dividing by multiples of 10. In this standard, students should reason about the magnitude of digits in a number. Students should be given opportunities to reason and analyze the relationships of numbers that they are working with. In the base-ten system, the value of each place is 10 times the value of the place to the immediate right. Because of this, multiplying by 10 yields a product in which each digit of the multiplicand is shifted one place to the left.  *Each of the 3 tens becomes a hundred and moves to the left. In the product, the 3 in the tens place of 30 is shifted one place to the left to represent 3 hundreds. In 300 divided by 10 the 3 is shifted one place to the right in the quotient to represent 3 tens.* (*Progressions for the CCSSM; Number and Operation in Base Ten*, CCSS Writing Team, April 2011, page 12) Example: How is the 2 in the number 582 similar to and different from the 2 in the number 528?

4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using **>, =**, and < symbols to record the results of comparisons.	This standard refers to various ways to write numbers. Students should have flexibility with the different number forms. Traditional expanded form is 285 = 200 + 80 + 5. Written form or number name is two hundred eighty-five. However, students should have opportunities to explore the idea that 285 could also be 28 tens plus 5 ones or 1 hundred, 18 tens, and 5 ones. To read numerals between 1,000 and 1,000,000, students need to understand the role of commas. Each sequence of three digits made by commas is read as hundreds, tens, and ones, followed by the name of the appropriate base-thousand unit (thousand, million, billion, trillion, etc.). Thus, 457,000 is read "four hundred fifty seven thousand." The same methods students used for comparing and rounding numbers in previous grades apply to these numbers, because of the uniformity of the base-ten system. (*Progressions for the CCSSM; Number and Operation in Base Ten*, CCSS Writing Team, April 2011, page 12) Students should also be able to compare two multi-digit whole numbers using appropriate symbols.
4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place.	This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding. Example: Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected? Continues on next page. Student 1 First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles. Student 2 First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40+20=60. 300-60 = 240, so we need about 240 more bottles.

Example:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel?
Some typical estimation strategies for this problem:

Student 1	Student 2	Student 3
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530.

Example:
Round 368 to the nearest hundred.
This will either be 300 or 400, since those are the two hundreds before and after 368.
Draw a number line, subdivide it as much as necessary, and determine whether 368 is closer to 300 or 400.
Since 368 is closer to 400, this number should be rounded to 400



Common Core Cluster

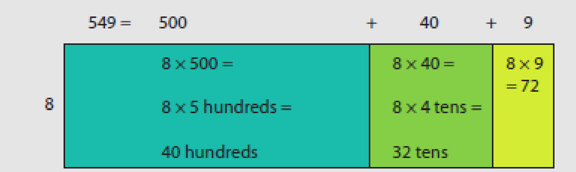
Use place value understanding and properties of operations to perform multi-digit arithmetic.

[1]Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **add, addend, sum, subtract, difference, equation, strategies, (properties)-rules about how numbers work, rectangular arrays, area model, multiply, divide, factor, product, quotient, reasonableness**

Common Core Standard	**Unpacking** What do these standards mean a child will know and be able to do?
4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.	Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract. This standard refers to fluency, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using a variety strategies such as the distributive property). This is the first grade level in which students are expected to be proficient at using the standard algorithm to add and subtract. However, other previously learned strategies are still appropriate for students to use. **Computation algorithm.** A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. **Computation strategy.** Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. (*Progressions for the CCSSM; Number and Operation in Base Ten,* CCSS Writing Team, April 2011, page 2) In mathematics, an algorithm is defined by its steps and not by the way those steps are recorded in writing. With this in mind, minor variations in methods of recording standard algorithms are acceptable. As with addition and subtraction, students should use methods they understand and can explain. Visual representations such as area and array diagrams that students draw and connect to equations and other written numerical work are useful for this purpose. By reasoning repeatedly about the connection between math drawings and written numerical work, students can come to see multiplication and division algorithms as abbreviations or

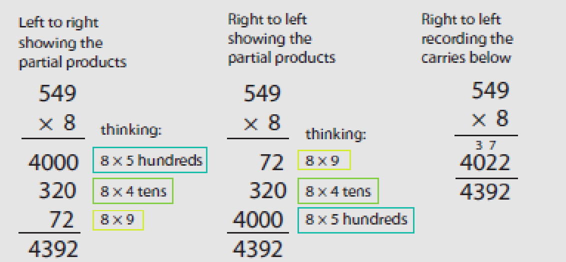
summaries of their reasoning about quantities.
Students can invent and use fast special strategies while also working towards understanding general methods and the standard algorithm.
One component of understanding general methods for multiplication is understanding how to compute products of one-digit numbers and multiples of 10, 100, and 1000. This extends work in Grade 3 on products of one-digit numbers and multiples of 10. We can calculate 6 x 700 by calculating 6 x 7 and then shifting the result to the left two places (by placing two zeros at the end to show that these are hundreds) because 6 groups of 7 hundred is 6 x 7 hundreds, which is 42 hundreds, or 4,200. Students can use this place value reasoning, which can also be supported with diagrams of arrays or areas, as they develop and practice using the patterns in relationships among products such as 6 x 7, 6 x 70, 6 x 700, and 6 x 7000. Products of 5 and even numbers, such as 5 x 4, 5 x 40, 5 x 400, 5 x 4000 and 4 x 5, 4 x 50, 4 x 500, 4 x 5000 might be discussed and practiced separately afterwards because they may seem at first to violate the patterns by having an “extra” 0 that comes from the one-digit product.



Each part of the region above corresponds to one of the terms in the computation below.

$$\begin{aligned} 8 \times 549 &= 8 \times (500 + 40 + 9) \\ &= 8 \times 500 + 8 \times 40 + 8 \times 9. \end{aligned}$$

This can also viewed as finding how many objects are in 8 groups of 549 objects, by finding the cardinalities of 8 groups of 500, 8 groups of 40, and 8 groups of 9, then adding them.



The first method proceeds from left to right, and the others from right to left. In the third method, the digits representing new units are written below the line rather than above 549, thus keeping the digits of the products close to each other, e.g., the 7 from $8 \times 9 = 72$ is written diagonally to the left of the 2 rather than above the 4 in 549.

(*Progressions for the CCSSM; Number and Operation in Base Ten*, CCSS Writing Team, April 2011, page 13)

When students begin using the standard algorithm their explanation may be quite lengthy. After much practice with using place value to justify their steps, they will develop fluency with the algorithm. Students should be able to explain why the algorithm works.

$$\begin{array}{r} 3892 \\ \underline{+\ 1567} \end{array}$$

Student explanation for this problem continued on the next page:

<table>
<tr>
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<td>

Two ones plus seven ones is nine ones.
Nine tens plus six tens is 15 tens.
I am going to write down five tens and think of the10 tens as one more hundred.(notates with a 1 above the hundreds column)
Eight hundreds plus five hundreds plus the extra hundred from adding the tens is 14 hundreds.
I am going to write the four hundreds and think of the 10 hundreds as one more 1000. (notates with a 1 above the thousands column)
Three thousands plus one thousand plus the extra thousand from the hundreds is five thousand.

$$\begin{array}{r} 3546 \\ \underline{-\ \ 928} \end{array}$$
Student explanation for this problem:

There are not enough ones to take 8 ones from 6 ones so I have to use one ten as 10 ones. Now I have 3 tens and 16 ones. (Marks through the 4 and notates with a 3 above the 4 and writes a 1 above the ones column to be represented as 16 ones.)
Sixteen ones minus 8 ones is 8 ones. (Writes an 8 in the ones column of answer.)
Three tens minus 2 tens is one ten. (Writes a 1 in the tens column of answer.)
There are not enough hundreds to take 9 hundreds from 5 hundreds so I have to use one thousand as 10 hundreds. (Marks through the 3 and notates with a 2 above it. (Writes down a 1 above the hundreds column.) Now I have 2 thousand and 15 hundreds.
Fifteen hundreds minus 9 hundreds is 6 hundreds. (Writes a 6 in the hundreds column of the answer).
I have 2 thousands left since I did not have to take away any thousands. (Writes 2 in the thousands place of answer.)

Students should know that it is mathematically possible to subtract a larger number from a smaller number but that their work with whole numbers does not allow this as the difference would result in a negative number.
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<td>4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td>
<td>
Students who develop flexibility in breaking numbers apart have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. Use of the standard algorithm for multiplication is an expectation in the 5th grade.
Another part of understanding general base-ten methods for multi-digit multiplication is understanding the role played by the distributive property. This allows numbers to be decomposed into base-ten units, products of the units to be computed, and then combined. By decomposing the factors into like base-ten units and applying the
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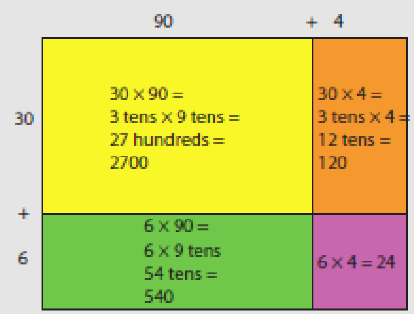
distributive property, multiplication computations are reduced to single-digit multiplications and products of numbers with multiples of 10, of 100, and of 1000. Students can connect diagrams of areas or arrays to numerical work to develop understanding of general base-ten multiplication methods. Computing products of two two-digit numbers requires using the distributive property several times when the factors are decomposed into base-ten units.

Example:

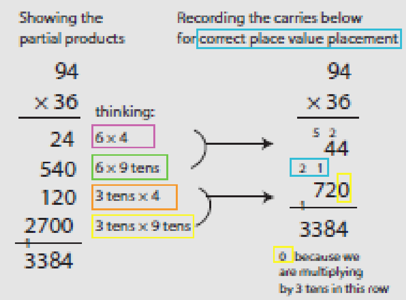
$$\begin{aligned} 36 \times 94 &= (30+6) \times (90+4) \\ &= (30+6) \times 90 + (30+6) \times 4 \\ &= 30 \times 90 + 6 \times 90 + 30 \times 4 + 6 \times 4. \end{aligned}$$

Computation of 36×94 connected with an area model



The products of like base-ten units are shown as parts of a rectangular region.

Computation of 36×94: Ways to record general methods



These proceed from right to left, but could go left to right. On the right, digits that represent newly composed tens and hundreds are written below the line instead of above 94. The digits 2 and 1 are surrounded by a blue box. The 1 from $30 \times 4 = 120$ is placed correctly in the hundreds place and the digit 2 from $30 \times 90 = 2700$ is placed correctly in the thousands place. If these digits had been placed above 94, they would be in incorrect places. Note that the 0 (surrounded by a yellow box) in the ones place of the second line of the method on the right is there because the whole line of digits is produced by multiplying by 30 (not 3).

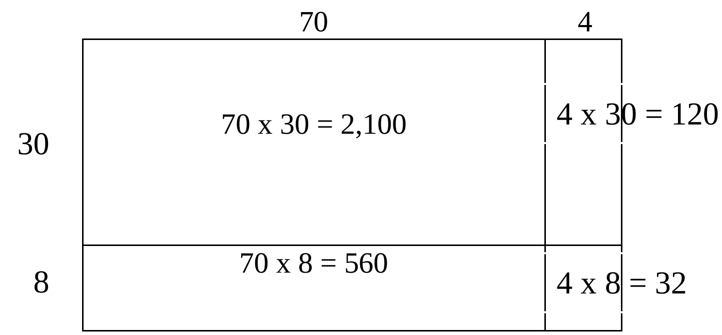
(*Progressions for the CCSSM; Number and Operation in Base Ten*, CCSS Writing Team, April 2011, page 14)

This standard calls for students to multiply numbers using a variety of strategies.

Example:
There are 25 dozen cookies in the bakery. What is the total number of cookies at the bakery?

Student 1	Student 2	Student 3
25 x12 I broke 12 up into 10 and 2 25 x 10 = 250 25 x 2 = 50 250 +50 = 300	25 x 12 I broke 25 up into 5 groups of 5 5 x 12 = 60 I have 5 groups of 5 in 25 60 x 5 = 300	25 x 12 I doubled 25 and cut 12 in half to get 50 x 6 50 x 6 = 300

Example:
What would an array area model of 74 x 38 look like?

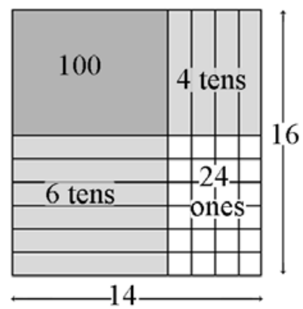


2,100 = 560 + 1,200 + 32 = 2,812

Example:
To illustrate 154 x 6 students use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the distributive property, 154 x 6 = (100 + 50 + 4) x 6 = (100 x 6) + (50 X 6) + (4 X 6) = 600 + 300 + 24 = 924.

The area model below shows the partial products.

14 x 16 = 224



100 + 40 + 60 + 24 = 224

Using the area model, students first verbalize their understanding:

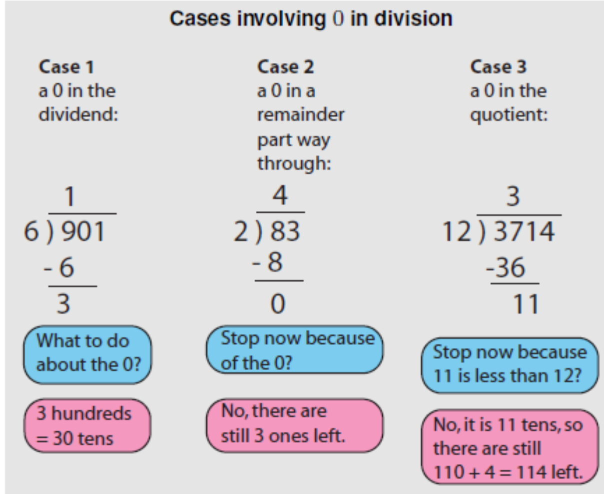
- 10 x 10 is 100
- 4 x 10 is 40
- 10 x 6 is 60, and
- 4 x 6 is 24.

They use different strategies to record this type of thinking.

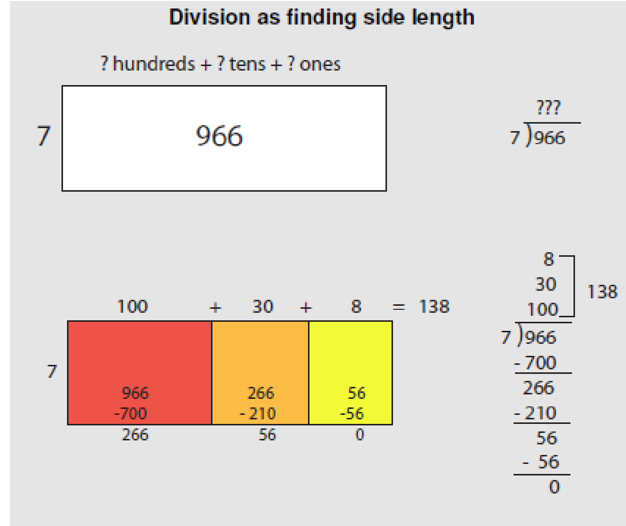
Students explain this strategy and the one below with base 10 blocks, drawings, or numbers.

$$\begin{array}{rl} 25 & \\ \underline{\times 24} & \\ 400 & (20 \times 20) \\ 100 & (20 \times 5) \\ 80 & (4 \times 20) \\ \underline{20} & (4 \times 5) \\ 600 & \end{array}$$

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context. General methods for computing quotients of multi-digit numbers and one-digit numbers rely on the same understandings as for multiplication, but cast in terms of division. One component is quotients of multiples of 10, 100, or 1000 and one-digit numbers. For example, 42 ÷ 6 is related to 420 ÷ 6 and 4200 ÷ 6. Students can draw on their work with multiplication and they can also reason that 4200 ÷ 6 means partitioning 42 hundreds into 6 equal groups, so there are 7 hundreds in each group. Another component of understanding general methods for multi-digit division computation is the idea of decomposing the dividend into like base-ten units and finding the quotient unit by unit, starting with the largest unit and continuing on to smaller units. As with multiplication, this relies on the distributive property. This can be viewed as finding the side length of a rectangle (the divisor is the length of the other side) or as allocating objects (the divisor is the number of groups). Multi-digit division requires working with remainders. In preparation for working with remainders, students can compute sums of a product and a number, such as 4 x 8 + 3. In multi-digit division, students will need to find the greatest multiple less than a given number. For example, when dividing by 6, the greatest multiple of 6 less than 50 is 6 x 8 = 48. Students can think of these "greatest multiples" in terms of putting objects into groups. For example, when 50 objects are shared among 6 groups, the largest whole number of objects that can be put in each group is 8, and 2 objects are left over. (Or when 50 objects are allocated into groups of 6, the largest whole number of groups that can be made is 8, and 2 objects are left over.) The equation 6 x 8 + 2 = 50 (or 8 x 6 + 2 = 50) corresponds with this situation. Cases involving 0 in division may require special attention. (*Progressions for the CCSSM; Number and Operation in Base Ten,* CCSS Writing Team, April 2011, page 14)

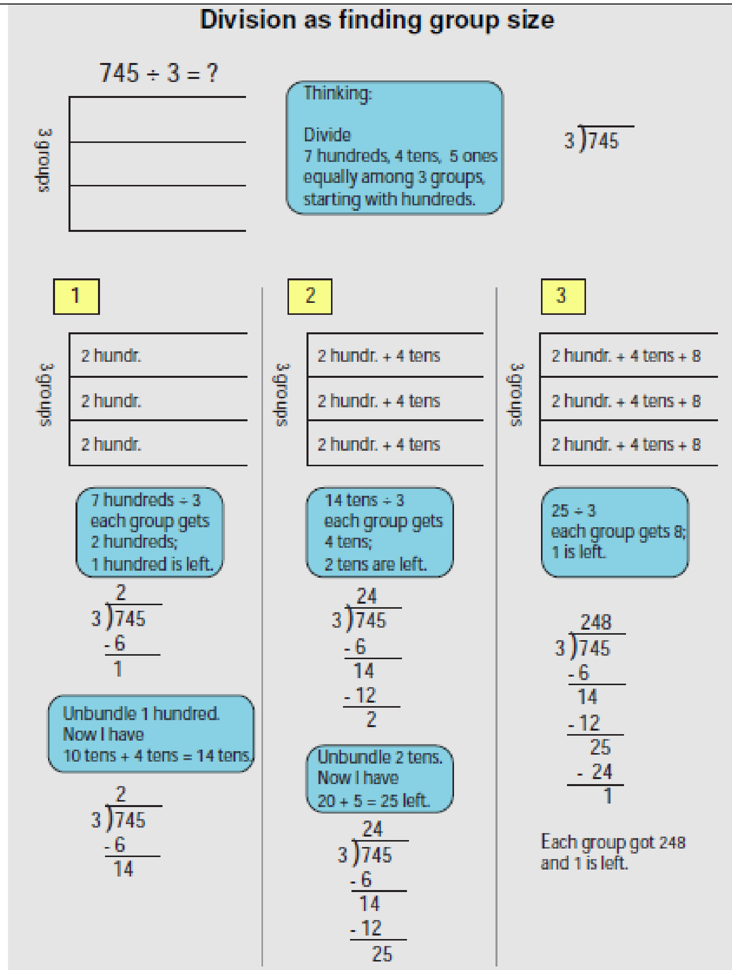


The focus of this standard is to build conceptual understanding of division. Although the tradition division algorithm may be introduced, students are not expected to master this algorithm until middle school.



$966 \div 7$ is viewed as finding the unknown side length of a rectangular region with area 966 square units and a side of length 7 units. The amount of hundreds is found, then tens, then ones. This yields a decomposition into three regions of dimensions 7 by 100, 7 by 30, and 7 by 8. It can be connected with the decomposition of 966 as $7 \times 100 + 7 \times 30 + 7 \times 8$. By the distributive property, this is $7 \times (100 + 30 + 8)$, so the unknown side length is 138. In the recording on the right, amounts of hundreds, tens, and ones are represented by numbers rather than by digits, e.g., 700 instead of 7.

(*Progressions for the CCSSM; Number and Operation in Base Ten*, CCSS Writing Team, April 2011, page 15)



$745 \div 3$ can be viewed as allocating 745 objects bundled in 7 hundreds, 4 tens, and 3 ones equally among 3 groups. In Step 1, the 2 indicates that each group got 2 hundreds, the 6 is the number of hundreds allocated, and the 1 is the number of hundreds not allocated. After Step 1, the remaining hundred is decomposed as 10 tens and combined with the 4 tens (in 745) to make 14 tens.

(*Progressions for the CCSSM; Number and Operation in Base Ten*, CCSS Writing Team, April 2011, page 15)

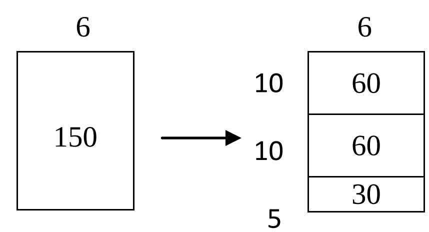
Example:
A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?

- Using Base 10 Blocks: Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.
- Using Place Value: 260 ÷ 4 = (200 ÷ 4) + (60 ÷ 4)
- Using Multiplication: 4 x 50 = 200, 4 x 10 = 40, 4 x 5 = 20; 50 + 10 + 5 = 65; so 260 ÷ 4 = 65

This standard calls for students to explore division through various strategies.
Example:
There are 592 students participating in Field Day. They are put into teams of 8 for the competition. How many teams get created?

Student 1	Student 2		Student 3
592 divided by 8 There are 70 8's in 560 592 - 560 = 32 There are 4 8's in 32 70 + 4 = 74	592 divided by 8 I know that 10 8's is 80 If I take out 50 8's that is 400 592 - 400 = 192 I can take out 20 more 8's which is 160 192 - 160 = 32 8 goes into 32 4 times I have none left I took out 50, then 20 more, then 4 more That's 74	592 -400 50 192 -160 20 32 -32 4 0	I want to get to 592 8 x 25 = 200 8 x 25 = 200 8 x 25 = 200 200 + 200 + 200 = 600 600 - 8 = 592 I had 75 groups of 8 and took one away, so there are 74 teams

Example:
Using an Open Array or Area Model
After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in the 5th grade.
Example: 150 ÷ 6



Students make a rectangle and write 6 on one of its sides. They express their understanding that they need to think of the rectangle as representing a total of 150.

1. Students think, 6 times what number is a number close to 150? They recognize that 6 x 10 is 60 so they record 10 as a factor and partition the rectangle into 2 rectangles and label the area aligned to the factor of 10 with 60. They express that they have only used 60 of the 150 so they have 90 left.
2. Recognizing that there is another 60 in what is left they repeat the process above. They express that they have used 120 of the 150 so they have 30 left.
3. Knowing that 6 x 5 is 30. They write 30 in the bottom area of the rectangle and record 5 as a factor.
4. Students express their calculations in various ways:

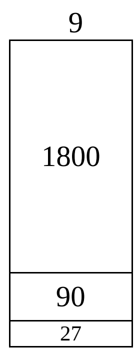
    a. 150   150 ÷ 6 = 10 + 10 + 5 = 25

    $$\begin{array}{rl} 150 & \\ \underline{-\ 60} & (6 \times 10) \\ 90 & \\ \underline{-\ 60} & (6 \times 10) \\ 30 & \\ \underline{-\ 30} & \underline{(6 \times 5)} \\ 0 & \end{array}$$

    b. 150 ÷ 6 = (60 ÷ 6) + (60 ÷ 6) + (30 ÷ 6) = 10 + 10 + 5 = 25

Example:

1917 ÷ 9



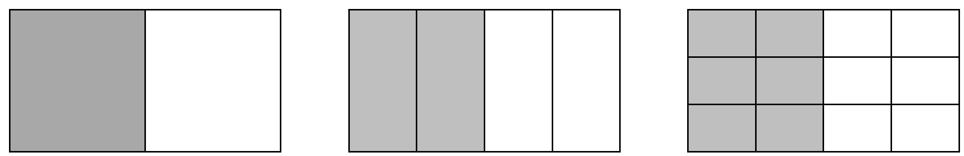
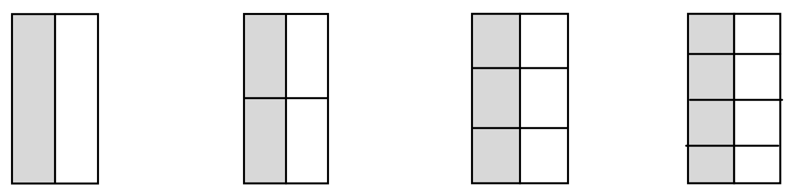
A student's description of his or her thinking may be:
I need to find out how many 9s are in 1917. I know that 200 x 9 is 1800. So if I use 1800 of the 1917, I have 117 left. I know that 9 x 10 is 90. So if I have 10 more 9s, I will have 27 left. I can make 3 more 9s. I have 200 nines, 10 nines and 3 nines. So I made 213 nines. 1917 ÷ 9 = 213.

Number and Operation – Fractions[1] 4.NF

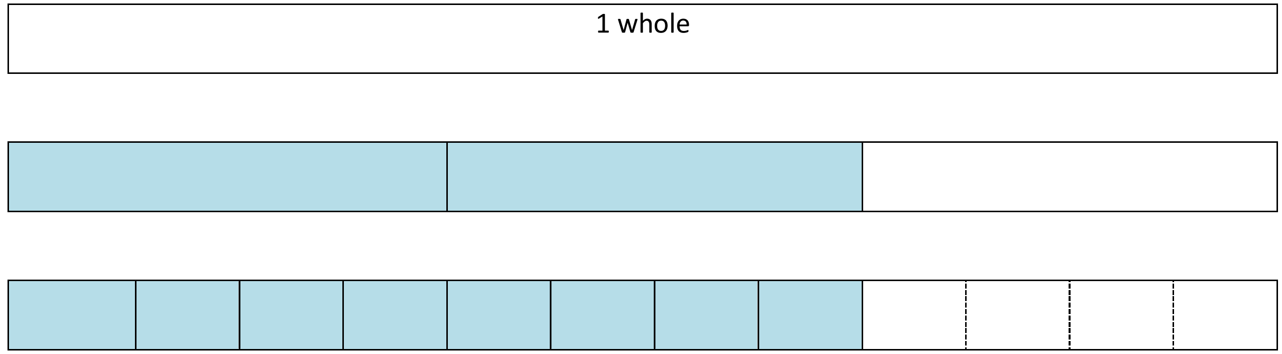
Common Core Cluster

Extend understanding of fraction equivalence and ordering.
Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **partition(ed), fraction, unit fraction, equivalent, expression, multiple, reason, denominator, numerator, comparison/compare, ‹, ›, =, benchmark fraction**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
4.NF.1 Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. [1]Grade 4 expectation in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.	This standard refers to visual fraction models. This includes area models, number lines or it could be a collection/set model. This standard extends the work in third grade by using additional denominators. (5, 10, 12 and 100) This standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number or by dividing a shaded region into various parts. Example:  **1/2 = 2/4 = 6/12** Students should begin to notice connections between the models and fractions in the way both the parts and wholes are counted and begin to generate a rule for writing equivalent fractions. 1/2 x 2/2 = 2/4.  $\frac{1}{2}$ $\frac{2}{4} = \frac{2 \times 1}{2 \times 2}$ $\frac{3}{6} = \frac{3 \times 1}{3 \times 2}$ $\frac{4}{8} = \frac{4 \times 1}{4 \times 2}$

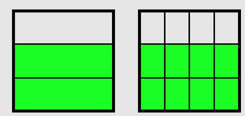
Example:



$$\frac{2}{3} \quad \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

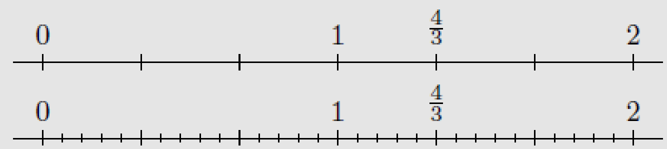
Example:

Using an area model to show that $\frac{2}{3} = \frac{4 \times 2}{4 \times 3}$

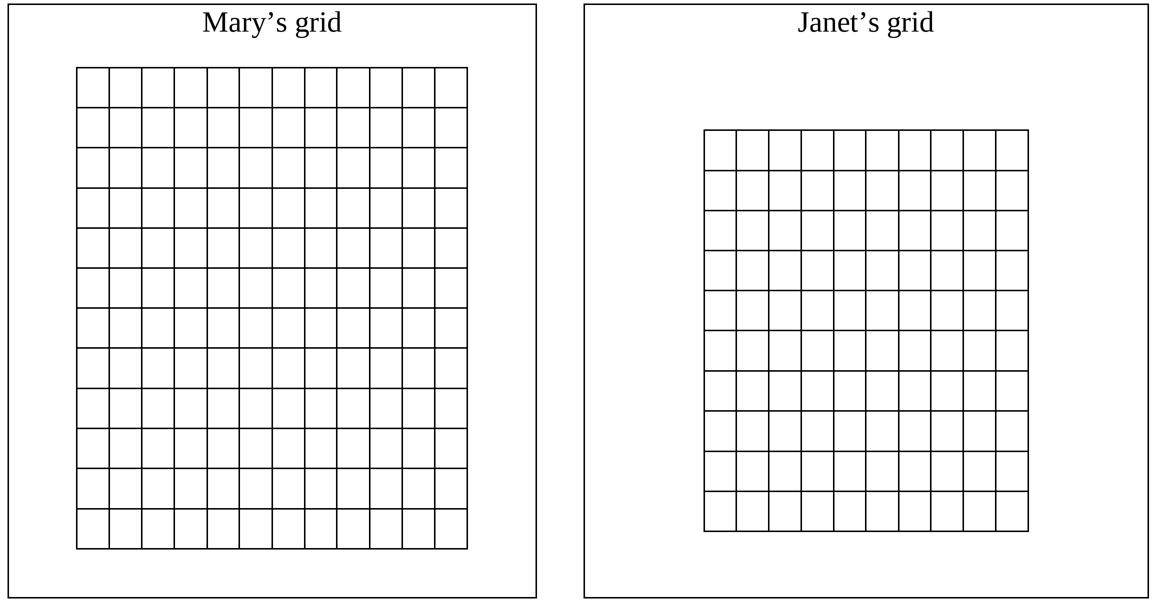


The whole is the square, measured by area. On the left it is divided horizontally into 3 rectangles of equal area, and the shaded region is 2 of these and so represents $\frac{2}{3}$. On the right it is divided into 4×3 small rectangles of equal area, and the shaded area comprises 4×2 of these, and so it represents $\frac{4 \times 2}{4 \times 3}$.

(*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 5)

	Using the number line to show that $\frac{4}{3} = \frac{5\times4}{5\times3}$  *$\frac{4}{3}$ is 4 parts when each part is $\frac{1}{3}$, and we want to see that this is also 5×4 parts when each part is $\frac{1}{5\times3}$. Divide each of the intervals of length $\frac{1}{3}$ into 5 parts of equal length. There are 5×3 parts of equal length in the unit interval, and $\frac{4}{3}$ is 5×4 of these. Therefore $\frac{4}{3} = \frac{5\times4}{5\times3} = \frac{20}{15}$.* (*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 5) There is **NO** mathematical reason why fractions must be written in simplified form, although it may be convenient to do so in some cases. Technology Connection: http://illuminations.nctm.org/activitydetail.aspx?id=80
4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	This standard calls students to compare fractions by creating visual fraction models or finding common denominators or numerators. Students' experiences should focus on visual fraction models rather than algorithms. When tested, models may or may not be included. Students should learn to draw fraction models to help them compare. Students must also recognize that they must consider the size of the whole when comparing fractions (ie, ½ and 1/8 of two medium pizzas is very different from ½ of one medium and 1/8 of one large). Example: Use patterns blocks. 1. If a red trapezoid is one whole, which block shows $\frac{1}{3}$? 2. If the blue rhombus is $\frac{1}{3}$, which block shows one whole? 3. If the red trapezoid is one whole, which block shows $\frac{2}{3}$? Example: Mary used a 12 x 12 grid to represent 1 and Janet used a 10 x 10 grid to represent 1. Each girl shaded grid squares to show $\frac{1}{4}$. How many grid squares did Mary shade? How many grid squares did Janet shade? Why did they need to shade different numbers of grid squares?

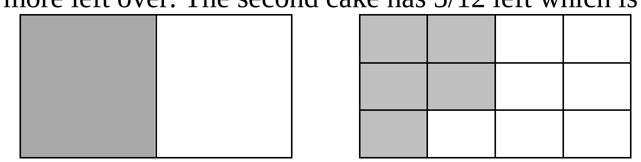
Possible solution: Mary shaded 36 grid squares; Janet shaded 25 grid squares. The total number of little squares is different in the two grids, so $\frac{1}{4}$ *of each total number is different.*



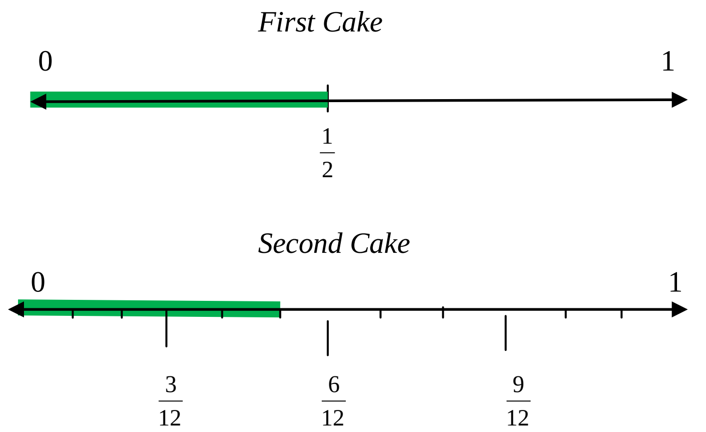
Example:

There are two cakes on the counter that are the same size. The first cake has ½ of it left. The second cake has 5/12 left. Which cake has more left?

Student 1
Area model:
The first cake has more left over. The second cake has 5/12 left which is smaller than ½.



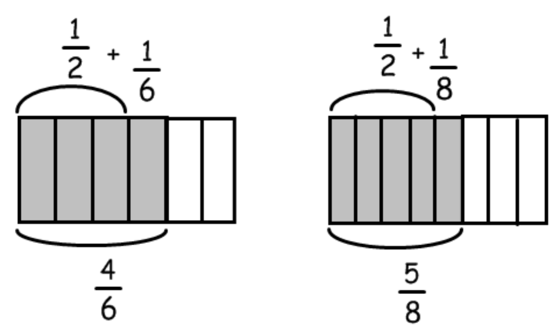
Student 2
Number Line model:



Student 3
verbal explanation:
I know that 6/12 equals ½. Therefore, the second cake which has 5/12 left is less than ½.

Example:

When using the benchmark of $\frac{1}{2}$ to compare $\frac{4}{6}$ and $\frac{5}{8}$, you could use diagrams such as these:



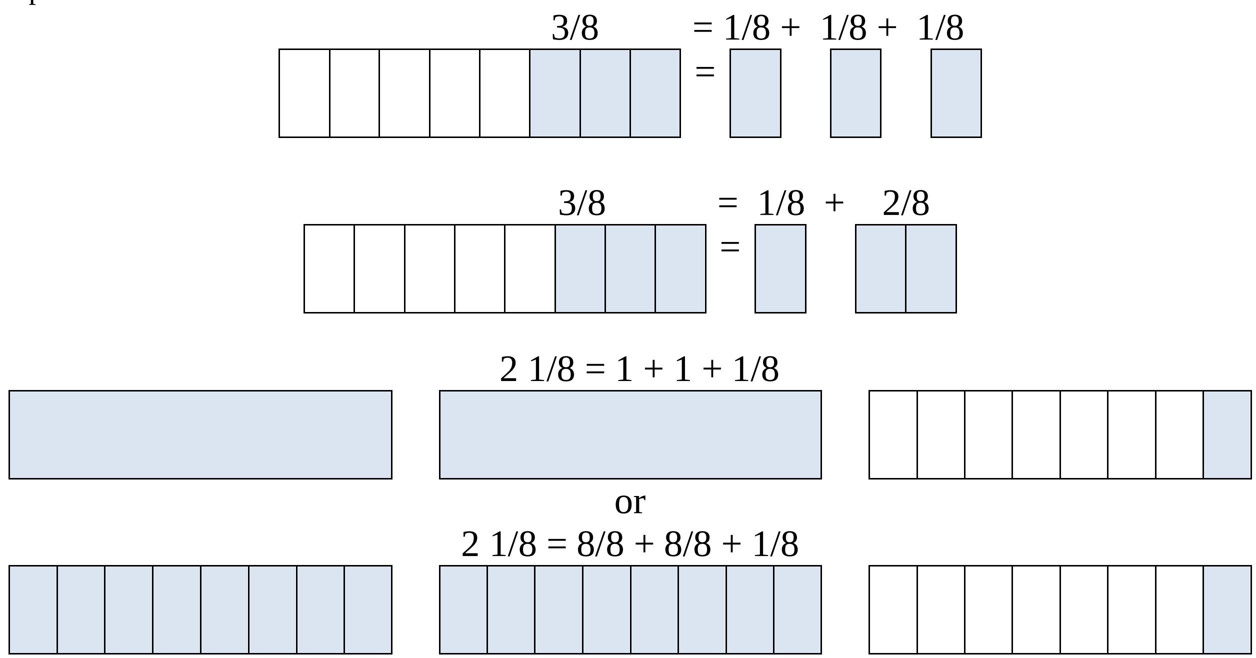
	$\frac{4}{6}$ is $\frac{1}{6}$ larger than $\frac{1}{2}$, while $\frac{5}{8}$ is $\frac{1}{8}$ larger than $\frac{1}{2}$. Since $\frac{1}{6}$ is greater than $\frac{1}{8}$, $\frac{4}{6}$ is the greater fraction. In fifth grade students who have learned about fraction multiplication can see equivalence as "multiplying by 1": $$\frac{7}{9}=\frac{7}{9}\times 1=\frac{7}{9}\times\frac{4}{4}=\frac{28}{36}$$ However, although a useful mnemonic device, this does **not constitute a valid argument at fourth grade**, since students have not yet learned fraction multiplication. (*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 6)

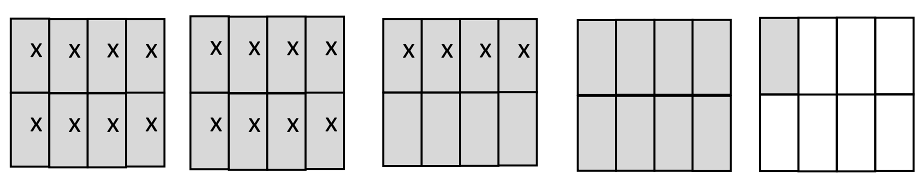
Common Core Cluster

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **operations, addition/joining, subtraction/separating, fraction, unit fraction, equivalent, multiple, reason, denominator, numerator, decomposing, mixed number,(properties)-rules about how numbers work, multiply, multiple,**

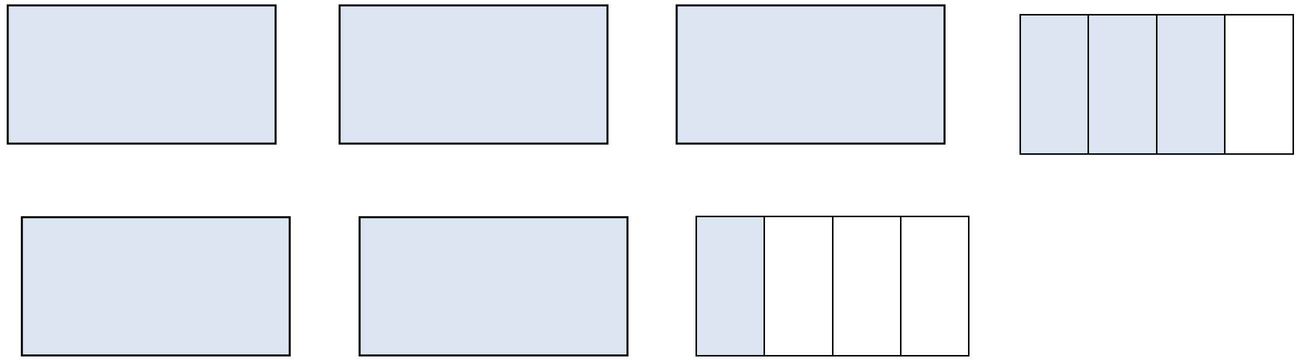
Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
4.NF.3 Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. **a.** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.	A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as 2/3, they should be able to join (compose) or separate (decompose) the fractions of the same whole. Example: 2/3 = 1/3 + 1/3 Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions. Students need multiple opportunities to work with mixed numbers and be able to decompose them in more than one way. Students may use visual models to help develop this understanding.

	Example: 1 ¼ - ¾ = ☐ 4/4 + ¼ = 5/4 5/4 – ¾ = 2/4 or ½ Example of word problem: Mary and Lacey decide to share a pizza. Mary ate 3/6 and Lacey ate 2/6 of the pizza. How much of the pizza did the girls eat together? Possible solution: The amount of pizza Mary ate can be thought of a 3/6 or 1/6 and 1/6 and 1/6. The amount of pizza Lacey ate can be thought of a 1/6 and 1/6. The total amount of pizza they ate is 1/6 + 1/6 + 1/6 + 1/6 + 1/6 or 5/6 of the whole pizza. Example: Five friends ordered 3 large sandwiches. John ate ¾ of a sandwich. Kim at ¼ of a sandwich. Ron ate ¾ of a sandwich. Sam ate 2/4 of a sandwich. How much sandwich is left? Explain your reasoning. (solution ¾ of a sandwich)
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*	Students should justify their breaking apart (decomposing) of fractions using visual fraction models. The concept of turning mixed numbers into improper fractions needs to be emphasized using visual fraction models. Example: 

	Similarly, converting an improper fraction to a mixed number is a matter of decomposing the fraction into a sum of a whole number and a number less than 1. Students can draw on their knowledge from third grade of whole numbers as fractions. Example, knowing that 1 = 3/3, they see: $\frac{5}{3} = \frac{3}{3} + \frac{2}{3} = 1 + \frac{2}{3} = 1\frac{2}{3}$ (*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 8)
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.	A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions. Example: Susan and Maria need 8 3/8 feet of ribbon to package gift baskets. Susan has 3 1/8 feet of ribbon and Maria has 5 3/8 feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not. The student thinks: I can add the ribbon Susan has to the ribbon Maria has to find out how much ribbon they have altogether. Susan has 3 1/8 feet of ribbon and Maria has 5 3/8 feet of ribbon. I can write this as 3 1/8 + 5 3/8. I know they have 8 feet of ribbon by adding the 3 and 5. They also have 1/8 and 3/8 which makes a total of 4/8 more. Altogether they have 8 4/8 feet of ribbon. 8 4/8 is larger than 8 3/8 so they will have enough ribbon to complete the project. They will even have a little extra ribbon left, 1/8 foot. Example: Trevor has 4 1/8 pizzas left over from his soccer party. After giving some pizza to his friend, he has 2 4/8 of a pizza left. How much pizza did Trevor give to his friend? Possible solution: Trevor had 4 1/8 pizzas to start. This is 33/8 of a pizza. The x's show the pizza he has left which is 2 4/8 pizzas or 20/8 pizzas. The shaded rectangles without the x's are the pizza he gave to his friend which is 13/8 or 1 5/8 pizzas. 

Mixed numbers are introduced for the first time in Fourth Grade. Students should have ample experiences of adding and subtracting mixed numbers where they work with mixed numbers or convert mixed numbers so that the numerator is equal to or greater than the denominator.

Example:
While solving the problem, 3 ¾ + 2 ¼ students could do the following:



Student 1
3 + 2 = 5 and ¾ + ¼ = 1 so 5+ 1 = 6

Student 2
3 ¾ + 2 = 5 ¾ so 5 ¾ + ¼ = 6

Student 3
3 ¾ = 15/4 and 2 ¼ = 9/4 so 15/4 + 9/4 = 24/4 = 6

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Fourth Grade students should be able to decompose and compose fractions with the same denominator. They add fractions with the same denominator.
Example:

$$\begin{aligned}\frac{7}{5}+\frac{4}{5} &= \overbrace{\frac{1}{5}+\cdots\frac{1}{5}}^{7}+\overbrace{\frac{1}{5}+\cdots\frac{1}{5}}^{4}\\ &= \frac{\overbrace{1+1+\cdots+1}^{7+4}}{5}\\ &= \frac{7+4}{5}.\end{aligned}$$

Using the understanding gained from work with whole numbers of the relationship between addition and subtraction, they also subtract fractions with the same denominator. For example, to subtract 5/6 from 17/6, they decompose.
Example:

$$\frac{12}{6}+\frac{5}{6}, \quad \text{so} \quad \frac{17}{6}-\frac{5}{6}=\frac{17-5}{6}=\frac{12}{6}=2$$

Students also compute sums of whole numbers and fractions, by representing the whole number as an equivalent fraction with the same denominator as the fraction.
Example:

$$7\frac{1}{5}=7+\frac{1}{5}=\frac{35}{5}+\frac{1}{5}=\frac{36}{5}.$$

Students use this method to add mixed numbers with like denominators. Converting a mixed number to a fraction should not be viewed as a separate technique to be learned by rote, but simply as a case of fraction addition.
(*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 6-7)
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<td>d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.</td>
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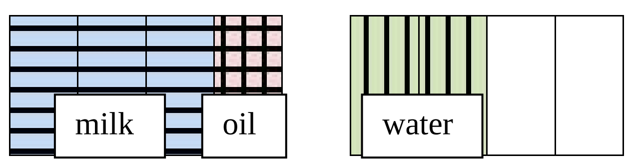
A cake recipe calls for you to use ¾ cup of milk, ¼ cup of oil, and 2/4 cup of water. How much liquid was needed to make the cake?



3/4 + 1/4 + 2/4 = 6/4 = 1 2/4
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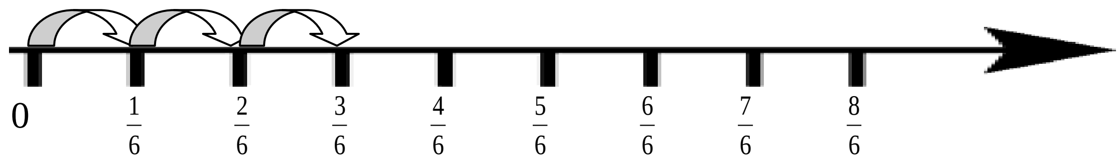
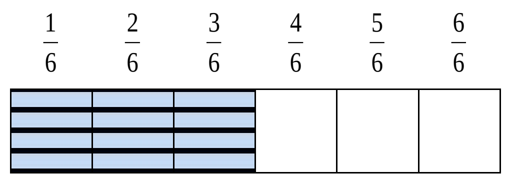
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction *a*/*b* as a multiple of 1/*b*.
For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).

This standard builds on students' work of adding fractions and extending that work into multiplication.
Example:
3/6 = 1/6 + 1/6 + 1/6 = 3 x (1/6)
Number line:



Area model:



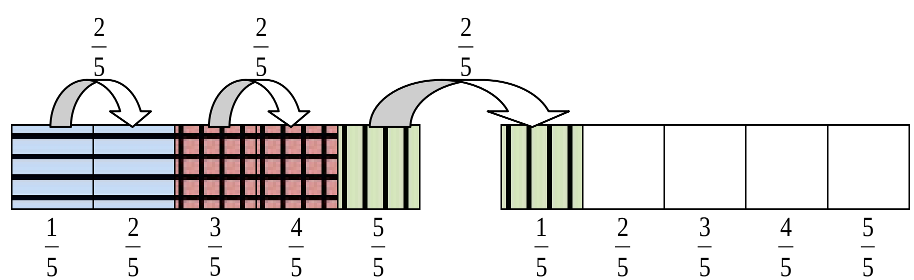
Students should see a fraction as the numerator times the unit fraction with the same denominator.
Example:

$$\frac{7}{5} = 7 \times \frac{1}{5}, \qquad \frac{11}{3} = 11 \times \frac{1}{3}$$

(*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 8)

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)

This standard extended the idea of multiplication as repeated addition. For example, 3 x (2/5) = 2/5 + 2/5 + 2/5 = 6/5 = 6 x (1/5). Students are expected to use and create visual fraction models to multiply a whole number by a fraction.



The same thinking, based on the analogy between fractions and whole numbers, allows students to give meaning to the product of whole number and a fraction.
Example:

$$3 \times \frac{2}{5} \quad \text{as} \quad \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{3 \times 2}{5} = \frac{6}{5}$$

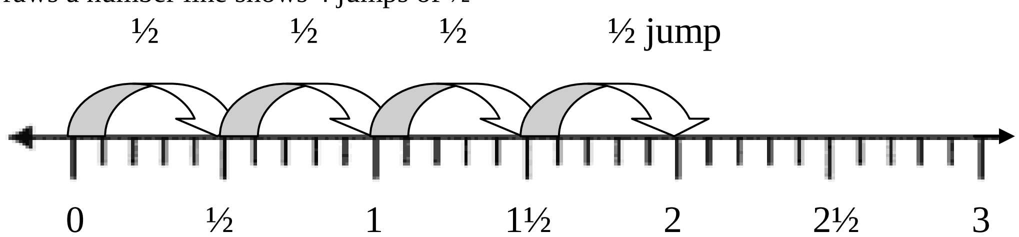
(*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 8)

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.
For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

When introducing this standard make sure student use visual fraction models to solve word problems related to multiplying a whole number by a fraction.

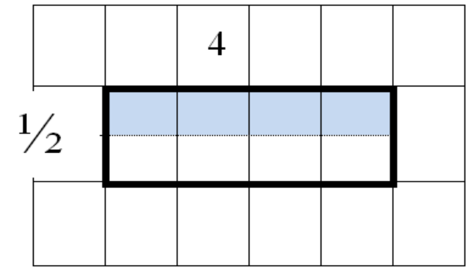
Example:
In a relay race, each runner runs ½ of a lap. If there are 4 team members how long is the race?

Student 1
Draws a number line shows 4 jumps of ½

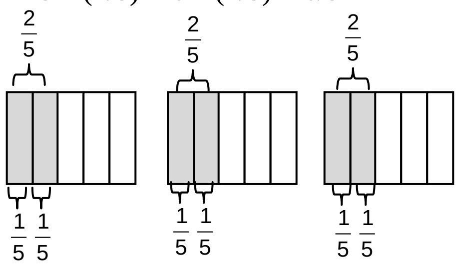


Student 2
Draws an area model showing 4 pieces of ½ joined together to equal 2.

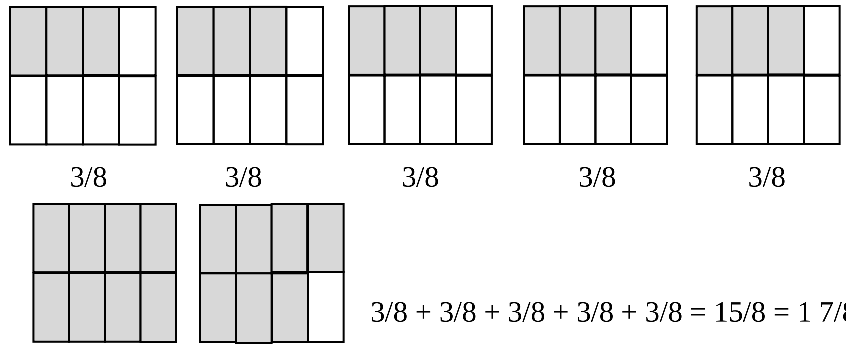
Student 3
Draws an area model representing 4 x ½ on a grid, dividing one row into ½ to represent the multiplier



Example:
Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns.
Examples: 3 x (2/5) = 6 x (1/5) = 6/5



If each person at a party eats 3/8 of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed? Between what two whole numbers does your answer lie?

	A student may build a fraction model to represent this problem:  3/8 + 3/8 + 3/8 + 3/8 + 3/8 = 15/8 = 1 7/8 Students solve word problems involving multiplication of a fraction by a whole number. Example: If a bucket holds 2 3/4 gallons and 43 buckets of water fill a tank, how much does the tank hold? The solution 43 x 2 3/4 gallons, one possible way to solve problem. $43 \times \left(2 + \frac{3}{4}\right) = 43 \times \frac{11}{4} = \frac{473}{4} = 118\frac{1}{4}$ gallons (*Progressions for the CCSSM, Number and Operation – Fractions,* CCSS Writing Team, August 2011, page 8)

Common Core Cluster

Understand decimal notation for fractions, and compare decimal fractions.

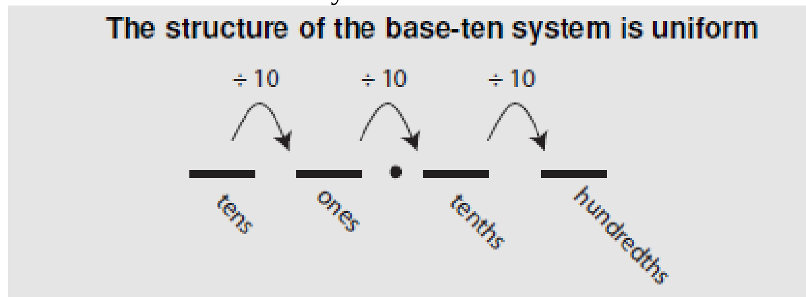
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are**: fraction, numerator, denominator, equivalent, reasoning, decimals, tenths, hundredths, multiplication, comparisons/compare, ‹, ›, =**

Common Core Standard	**Unpacking** What do these standards mean a child will know and be able to do?
4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and	This standard continues the work of equivalent fractions by having students change fractions with a 10 in the denominator into equivalent fractions that have a 100 in the denominator. In order to prepare for work with decimals (4.NF.6 and 4.NF.7), experiences that allow students to shade decimal grids (10x10 grids) can support this work. Student experiences should focus on working with grids rather than algorithms. Students can also use base ten blocks and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.

100.[2]
For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.

[2] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

Students in fourth grade work with fractions having denominators 10 and 100. Because it involves partitioning into 10 equal parts and treating the parts as numbers called one tenth and one hundredth, work with these fractions can be used as preparation to extend the base-ten system to non-whole numbers.

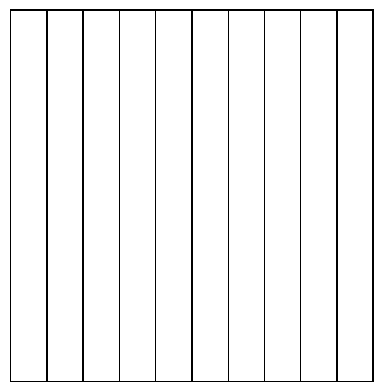


(*Progressions for the CCSSM; Number and Operation in Base Ten*, CCSS Writing Team, April 2011, page 12)
This work in fourth grade lays the foundation for performing operations with decimal numbers in fifth grade.

Example:

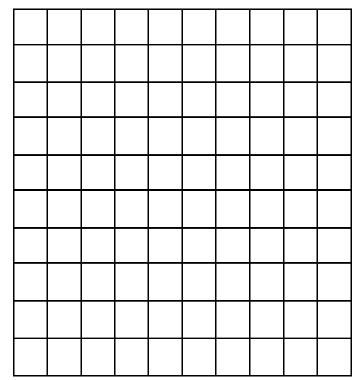
Ones	.	Tenths	Hundredths

Tenths Grid



.3 = 3 tenths = 3/10

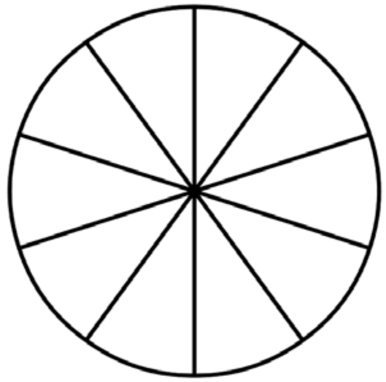
Hundredths Grid



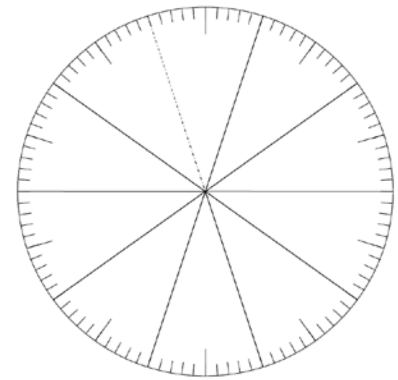
.30 = 30 hundredths = 30/100

Example:
Represent 3 tenths and 30 hundredths on the models below.

10ths circle



100ths circle



4.NF.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

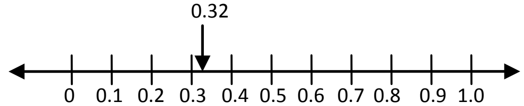
Decimals are introduced for the first time in fourth grade. Students should have ample opportunities to explore and reason about the idea that a number can be represented as both a fraction and a decimal.

Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say 32/100 as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below.

Hundreds	Tens	Ones	●	Tenths	Hundredths
			●	3	2

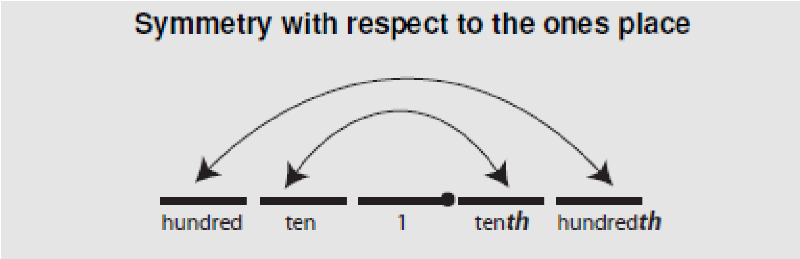
Students use the representations explored in 4.NF.5 to understand 32/100 can be expanded to 3/10 and 2/100.

Students represent values such as 0.32 or 32/100 on a number line. 32/100 is more than 30/100 (or 3/10) and less than 40/100 (or 4/10). It is closer to 30/100 so it would be placed on the number line near that value.

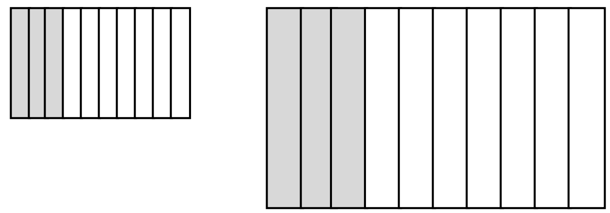


4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are

Students should reason that comparisons are only valid when they refer to the same whole. Visual models include area models, decimal grids, decimal circles, number lines, and meter sticks.

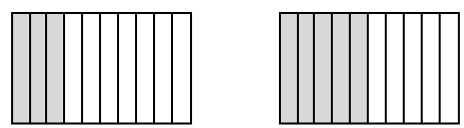
valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	The decimal point is used to signify the location of the ones place, but its location may suggest there should be a “oneths" place to its right in order to create symmetry with respect to the decimal point. However, because one is the basic unit from which the other base ten units are derived, the symmetry occurs instead with respect to the ones place. Ways of reading decimals aloud vary. Mathematicians and scientists often read 0.15 aloud as “zero point one five" or “point one five." (Decimals smaller than one may be written with or without a zero before the decimal point.) Decimals with many non-zero digits are more easily read aloud in this manner. (For example, the number π, which has infinitely many non-zero digits, begins 3.1415 . . ....) Other ways to read 0.15 aloud are “1 tenth and 5 hundredths” and “15 hundredths,” just as 1,500 is sometimes read “15 hundred” or “1 thousand, 5 hundred.” Similarly, 150 is read “one hundred and fifty” or “a hundred fifty” and understood as 15 tens, as 10 tens and 5 tens, and as 100 + 50. Just as 15 is understood as 15 ones and as 1 ten and 5 ones in computations with whole numbers, 0.15 is viewed as 15 hundredths and as 1 tenth and 5 hundredths in computations with decimals. It takes time to develop understanding and fluency with the different forms. Layered cards for decimals can help students become fluent with decimal equivalencies such as three tenths is thirty hundredths.  (*Progressions for the CCSSM; Number and Operation in Base Ten,* CCSS Writing Team, April 2011, page 12-13)

Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases. Each of the models below shows 3/10 but the whole on the right is much bigger than the whole on the left. They are both 3/10 but the model on the right is a much larger quantity than the model on the left.



When the wholes are the same, the decimals or fractions can be compared.

Example:
Draw a model to show that 0.3 < 0.5. (Students would sketch two models of approximately the same size to show the area that represents three-tenths is smaller than the area that represents five-tenths.



Measurement and Data 4.MD

Common Core Cluster

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, metric, customary, convert/conversion, relative size, liquid volume, mass, length, distance, kilometer (km), meter (m), centimeter (cm), millimeter (mm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), time, hour, minute, second, equivalent, operations, add, subtract, multiply, divide, fractions, decimals, area, perimeter**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*	The units of measure that have not been addressed in prior years are cups, pints, quarts, gallons, pounds, ounces, kilometers, millimeter, milliliters, and seconds. Students' prior experiences were limited to measuring length, mass (metric and customary systems), liquid volume (metric only), and elapsed time. Students did not convert measurements. Students develop benchmarks and mental images about a meter (e.g., about the height of a tall chair) and a kilometer (e.g., the length of 10 football fields including the end zones, or the distance a person might walk in about 12 minutes), and they also understand that "kilo" means a thousand, so 3000 m is equivalent to 3 km. Expressing larger measurements in smaller units within the metric system is an opportunity to reinforce notions of place value. There are prefixes for multiples of the basic unit (meter or gram), although only a few (kilo-, centi-, and milli-) are in common use. Tables such as the one below are an opportunity to develop or reinforce place value concepts and skills in measurement activities. Relating units within the metric system is another opportunity to think about place value. For example, students might make a table that shows measurements of the same lengths in centimeters and meters. Relating units within the traditional system provides an opportunity to engage in mathematical practices, especially "look for and make use of structure" and "look for and express regularity in repeated reasoning" For example, students might make a table that shows measurements of the same lengths in feet and inches. (*Progressions for the CCSSM*, *Geometric Measurement*, CCSS Writing Team, June 2012, page20)

Super- or subordinate unit	Length in terms of basic unit
kilometer	10^3 or 1000 meters
hectometer	10^2 or 100 meters
decameter	10^1 or 10 meters
meter	1 meter
decimeter	10^{-1} or $\frac{1}{10}$ meters
centimeter	10^{-2} or $\frac{1}{100}$ meters
millimeter	10^{-3} or $\frac{1}{1000}$ meters

Note the similarity to the structure of base-ten units and U.S. currency (see illustrations on p. 12 of the Number and Operations in Base Ten Progression).

Centimeter and meter equivalences

cm	m
100	1
200	2
300	3
500	
1000	

Foot and inch equivalences

feet	inches
0	0
1	12
2	24
3	

(*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 20)

	Students need ample opportunities to become familiar with these new units of measure and explore the patterns and relationships in the conversion tables that they create. Students may use a two-column chart to convert from larger to smaller units and record equivalent measurements. They make statements such as, if one foot is 12 inches, then 3 feet has to be 36 inches because there are 3 groups of 12. Example: Customary length conversion table Yards / Feet: 1 / 3; 2 / 6; 3 / 9; ***n*** / ***n*** x 3 Foundational understandings to help with measure concepts: Understand that larger units can be subdivided into equivalent units (partition). Understand that the same unit can be repeated to determine the measure (iteration). Understand the relationship between the size of a unit and the number of units needed (compensatory principal). These Standards do not differentiate between weight and mass. Technically, mass is the amount of matter in an object. Weight is the force exerted on the body by gravity. On the earth's surface, the distinction is not important (on the moon, an object would have the same mass, would weigh less due to the lower gravity). (*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 2)
4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	This standard includes multi-step word problems related to expressing measurements from a larger unit in terms of a smaller unit (e.g., feet to inches, meters to centimeter, and dollars to cents). Students should have ample opportunities to use number line diagrams to solve word problems. Example: Charlie and 10 friends are planning for a pizza party. They purchased 3 quarts of milk. If each glass holds 8oz will everyone get at least one glass of milk? possible solution: Charlie plus 10 friends = 11 total people 11 people x 8 ounces (glass of milk) = 88 total ounces 1 quart = 2 pints = 4 cups = 32 ounces Therefore 1 quart = 2 pints = 4 cups = 32 ounces 2 quarts = 4 pints = 8 cups = 64 ounces 3 quarts = 6 pints = 12 cups = 96 ounces

Yards	Feet
1	3
2	6
3	9
n	***n*** x 3

If Charlie purchased 3 quarts (6 pints) of milk there would be enough for everyone at his party to have at least one glass of milk. If each person drank 1 glass then he would have 1- 8 oz glass or 1 cup of milk left over.

Additional Examples with various operations:
Division/fractions: Susan has 2 feet of ribbon. She wants to give her ribbon to her 3 best friends so each friend gets the same amount. How much ribbon will each friend get?
Students may record their solutions using fractions or inches. (The answer would be 2/3 of a foot or 8 inches. Students are able to express the answer in inches because they understand that 1/3 of a foot is 4 inches and 2/3 of a foot is 2 groups of 1/3.)

Addition: Mason ran for an hour and 15 minutes on Monday, 25 minutes on Tuesday, and 40 minutes on Wednesday. What was the total number of minutes Mason ran?

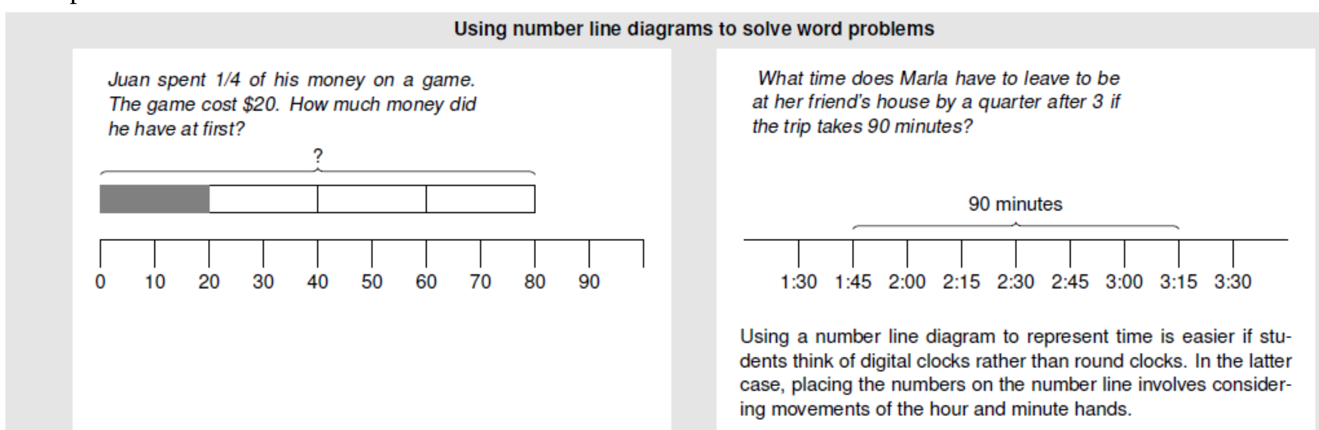
Subtraction: A pound of apples costs $1.20. Rachel bought a pound and a half of apples. If she gave the clerk a $5.00 bill, how much change will she get back?

Multiplication: Mario and his 2 brothers are selling lemonade. Mario brought one and a half liters, Javier brought

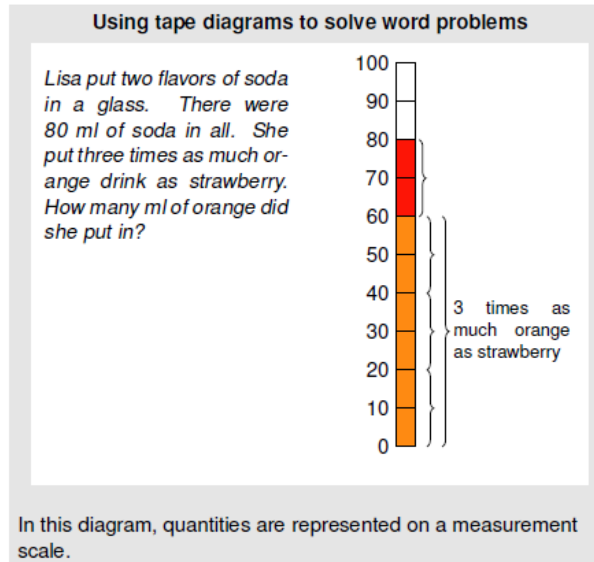
2 liters, and Ernesto brought 450 milliliters. How many total milliliters of lemonade did the boys have?

Number line diagrams that feature a measurement scale can represent measurement quantities. Examples include: ruler, diagram marking off distance along a road with cities at various points, a timetable showing hours throughout the day, or a volume measure on the side of a container.

Example:

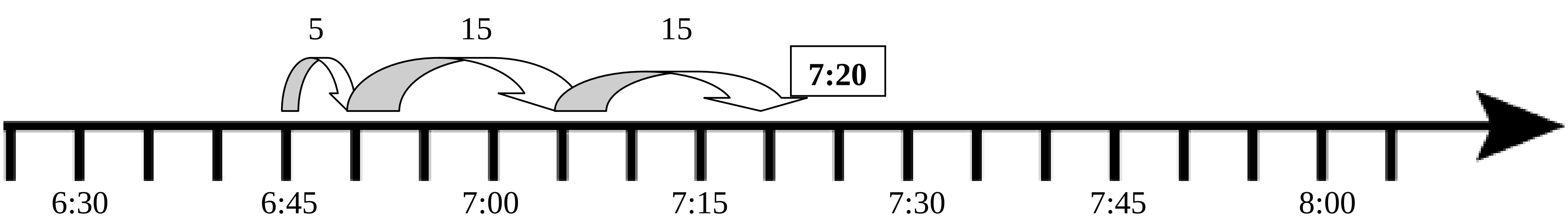


Students also combine competencies from different domains as they solve measurement problems using all four arithmetic operations, addition, subtraction, multiplication, and division.
Example: “How many liters of juice does the class need to have at least 35 cups if each cup takes 225 ml?”
Students may use tape or number line diagrams for solving such problems.
Example:



(*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 20)

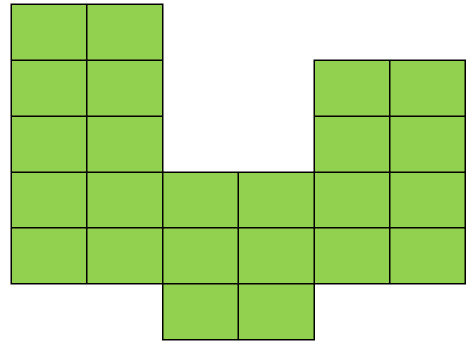
Example:
Tonya wakes up at 6:45 a.m. It takes her 5 minutes to shower, 15 minutes to get dressed, and 15 minutes to eat breakfast. What time will she be ready for school?



4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*	Based on work in third grade students learn to consider perimeter and area of rectangles. Fourth graders multiplication, spatially structuring arrays, and area, they abstract the formula for the area of a rectangle $A = l \times w$.• • The formula is a generalization of the understanding, that, given a unit of length, a rectangle whose sides have length *w* units and *l* units, can be partitioned into *w* rows of unit squares with *l* squares in each row. The product *l* x *w* gives the number of unit squares in the partition, thus the area measurement is *l* x *w* square units. These square units are derived from the length unit. Students generate and discuss advantages and disadvantages of various formulas for the perimeter length of a rectangle that is *l* units by *w* units. • • For example, $P = 2l + 2w$ has two multiplications and one addition, but $P = 2(l + w)$, which has one addition and one multiplication, involves fewer calculations. The latter formula is also useful when generating all possible rectangles with a given perimeter. The length and width vary across all possible pairs whose sum is half of the perimeter (e.g., for a perimeter of 20, the length and width are all of the pairs of numbers with sum 10). Giving verbal summaries of these formulas is also helpful. For example, a verbal summary of the basic formula, $P = l + w + l + w$, is “add the lengths of all four sides.” Specific numerical instances of other formulas or mental calculations for the perimeter of a rectangle can be seen as examples of the properties of operations, e.g., $2l + 2w = 2(l + w)$ illustrates the distributive property. Perimeter problems often give only one length and one width, thus remembering the basic formula can help to prevent the usual error of only adding one length and one width. The formula $P = 2\ (l \times w)$ emphasizes the step of multiplying the total of the given lengths by 2. Students can make a transition from showing all length units along the sides of a rectangle or all area units within by drawing a rectangle showing just parts of these as a reminder of which kind of unit is being used. Writing all of the lengths around a rectangle can also be useful. Discussions of formulas such as $P = 2l + 2w$, can note that unlike area formulas, perimeter formulas combine length measurements to yield a length measurement. Such abstraction and use of formulas underscores the importance of distinguishing between area and perimeter in Grade 3 and maintaining the distinction in Grade 4 and later grades, where rectangle perimeter and area problems may get more complex and problem solving can benefit from knowing or being able to rapidly remind oneself of how to find an area or perimeter. By repeatedly reasoning about how to calculate areas and perimeters of rectangles, students can come to see area and perimeter formulas as summaries of all such calculations. (*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 21)

Example:
Mr. Rutherford is covering the miniature golf course with an artificial grass. How many 1-foot squares of carpet will he need to cover the entire course?

1-foot square
of carpet



Students learn to apply these understandings and formulas to the solution of real-world and mathematical problems.
Example: A rectangular garden has as an area of 80 square feet. It is 5 feet wide. How long is the garden?
Here, specifying the area and the width creates an unknown factor problem. Similarly, students could solve perimeter problems that give the perimeter and the length of one side and ask the length of the adjacent side.

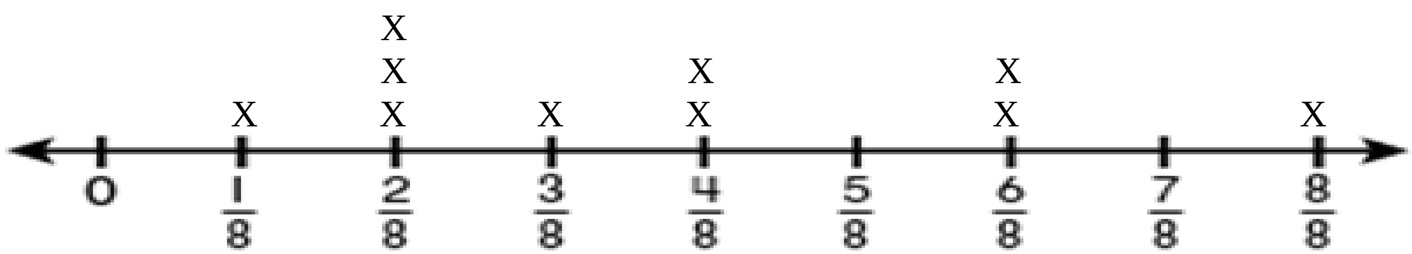
Students should be challenged to solve multistep problems.
Example: A plan for a house includes rectangular room with an area of 60 square meters and a perimeter of 32 meters. What are the length and the width of the room?
In fourth grade and beyond, the mental visual images for perimeter and area from third grade can support students in problem solving with these concepts. When engaging in the mathematical practice of reasoning abstractly and quantitatively in work with area and perimeter, students think of the situation and perhaps make a drawing. Then they recreate the "formula" with specific numbers and one unknown number as a situation equation for this particular numerical situation. "Apply the formula" does **not** mean write down a memorized formula and put in known values because in fourth grade students do not evaluate expressions (they begin this type of work in Grade 6). In fourth grade, working with perimeter and area of rectangles is still grounded in specific visualizations and numbers. These numbers can now be any of the numbers used in fourth grade (for addition and subtraction for perimeter and for multiplication and division for area). By repeatedly reasoning about constructing situation equations for perimeter and area involving specific numbers and an unknown number, students will build a foundation for applying area, perimeter, and other formulas by substituting specific values for the variables in later grades. (*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 22)

Common Core Cluster

Represent and interpret data.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **data, line plot, length, fractions**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*	This standard provides a context for students to work with fractions by measuring objects to an eighth of an inch. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot. Example: Students measured objects in their desk to the nearest ½, ¼, or 1/8 inch. They displayed their data collected on a line plot. How many object measured ¼ inch? ½ inch? If you put all the objects together end to end what would be the total length of **all** the objects.



Common Core Cluster

Geometric measurement: understand concepts of angle and measure angles.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, point, end point, geometric shapes, ray, angle, circle, fraction, intersect, one-degree angle, protractor, decomposed, addition, subtraction, unknown, obtuse, acute**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?

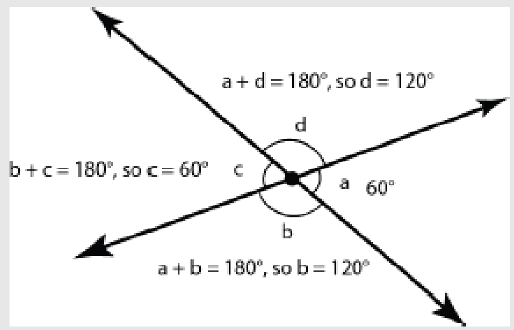
4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.	This standard brings up a connection between angles and circular measurement (360 degrees). Angle measure is a "turning point" in the study of geometry. Students often find angles and angle measure to be difficult concepts to learn, but that learning allows them to engage in interesting and important mathematics. An *angle* is the union of two rays, *a* and *b*, with the same initial point *P*. The rays can be made to coincide by rotating one to the other about *P*; this rotation determines the size of the angle between *a* and *b*. The rays are sometimes called the *sides* of the angles. Another way of saying this is that each ray determines a direction and the angle size measures the change from one direction to the other. Angles are measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and degrees are the unit used to measure angles in elementary school. A full rotation is thus 360º An *obtuse angle* is an angle with measures greater than 90º and less than 180º. An *acute angle* is an angle with measure less than 90º. Two angles are called *complementary* if their measurements have the sum of 90º. Two angles are called *supplementary* if their measurements have the sum of 180º. Two angles with the same vertex that overlap only at a boundary (i.e., share a side) are called *adjacent angles*. These terms may come up in classroom discussion, they will not be tested. This concept is developed thoroughly in middle school (7th grade). Like length, area, and volume, angle measure is additive: The sum of the measurements of *adjacent angles* is the measurement of the angle formed by their union. This leads to other important properties. If a right angle is decomposed into two adjacent angles, the sum is 90º, thus they are complementary. Two adjacent angles that compose a "straight angle" of 180º must be supplementary.

An angle

name	measurement
right angle	90°
straight angle	180°
acute angle	between 0 and 90°
obtuse angle	between 90° and 180°
reflex angle	between 180° and 360°

(*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 23)

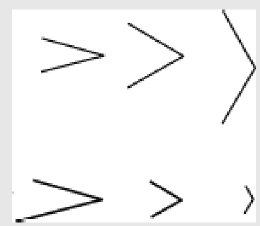
Angles created by the intersection of two lines



When two lines intersect, they form four angles. If the measurement of one is known (e.g., angle *a* is 60°), the measurement of the other three can be determined.

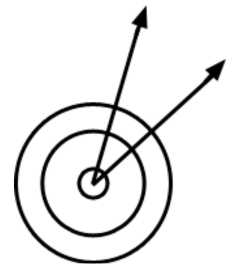
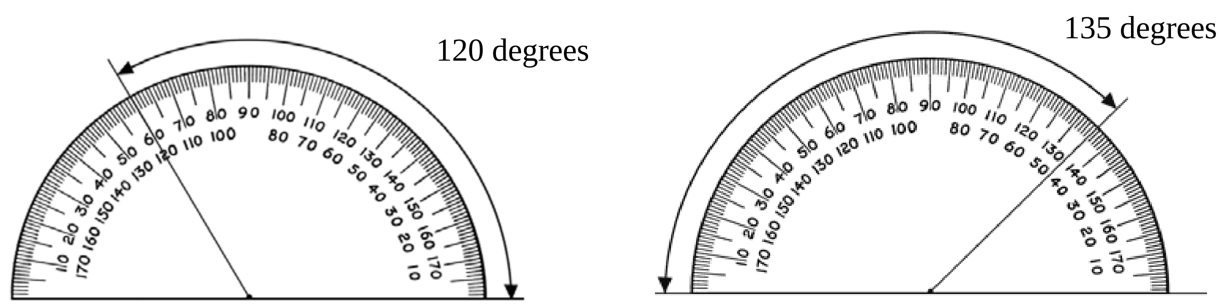
(*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 23)

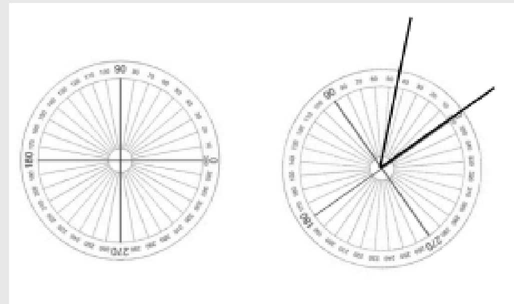
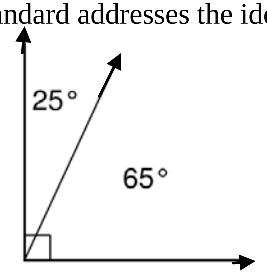
Two representations of three angles



Initially, some students may correctly compare angle sizes only if all the line segments are the same length (as shown in the top row). If the lengths of the line segments are different (as shown in the bottom row), these students base their judgments on the lengths of the segments, the distances between their endpoints, or even the area of the triangles determined by the drawn arms. They believe that the angles in the bottom row decrease in size from left to right, although they have, respectively, the same angle measurements as those in the top row.

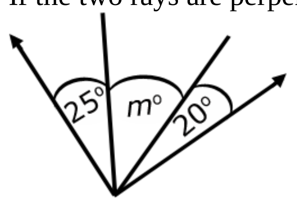
(*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 23)

b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	The diagram below will help students understand that an angle measurement is not related to an area since the area between the 2 rays is different for both circles yet the angle measure is the same.  This standard calls for students to explore an angle as a series of “one-degree turns.” A water sprinkler rotates one-degree at each interval. If the sprinkler rotates a total of 100º, how many one-degree turns has the sprinkler made?
4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles. They transfer their understanding that a 360º rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90º and 180º. They extend this understanding and recognize and sketch angles that measure approximately 45º and 30º. They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular). Students should measure angles and sketch angles  As with all measureable attributes, students must first recognize the attribute of angle measure, and distinguish it from other attributes. As with other concepts students need varied examples and explicit discussions to avoid

	learning limited ideas about measuring angles (e.g., misconceptions that a right angle is an angle that points to the right, or two right angles represented with different orientations are not equal in measure). If examples and tasks are not varied, students can develop incomplete and inaccurate notions. For example, some come to associate all slanted lines with 45º measures and horizontal and vertical lines with measures of 90º. Others believe angles can be “read off” a protractor in “standard” position, that is, a base is horizontal, even if neither ray of the angle is horizontal. Measuring and then sketching many angles with no horizontal or vertical ray perhaps initially using circular 360º protractors can help students avoid such limited conceptions. (*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 23) **A 360° protractor and its use**  The figure on the right shows a protractor being used to measure a 45° angle. The protractor is placed so that one side of the angle lies on the line corresponding to 0° on the protractor and the other side of the angle is located by a clockwise rotation from that line. (*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 23)
4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	This standard addresses the idea of decomposing (breaking apart) an angle into smaller parts.  Example: A lawn water sprinkler rotates 65 degress and then pauses. It then rotates an additional 25 degrees. What is the

total degree of the water sprinkler rotation? To cover a full 360 degrees how many times will the water sprinkler need to be moved?
If the water sprinkler rotates a total of 25 degrees then pauses. How many 25 degree cycles will it go through for the rotation to reach at least 90 degrees?

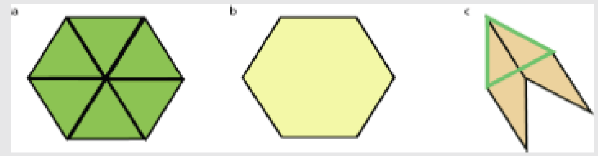
Example:
If the two rays are perpendicular, what is the value of m?



Example:
Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30º. What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4?

Students can develop more accurate and useful angle and angle measure concepts if presented with angles in a variety of situations. They learn to find the common features of superficially different situations such as turns in navigation, slopes, bends, corners, and openings. With guidance, they learn to represent an angle in any of these contexts as two rays, even when both rays are not explicitly represented in the context; for example, the horizontal or vertical in situations that involve slope (e.g., roads or ramps), or the angle determined by looking up from the horizon to a tree or mountain-top. Eventually they abstract the common attributes of the situations as angles (which are represented with rays and a vertex,) and angle measurements.

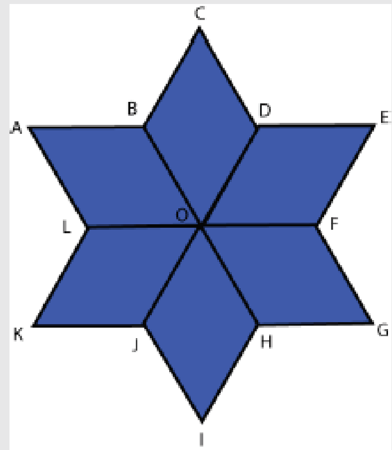
Determining angles in pattern blocks

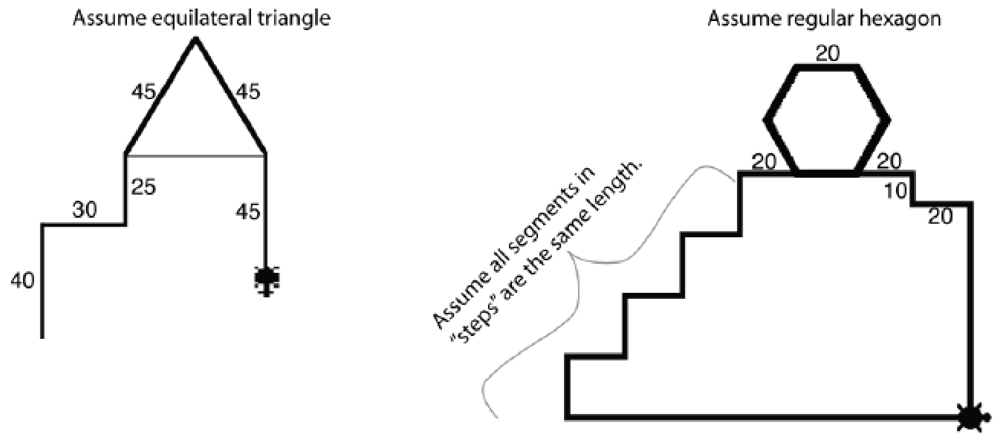


Students might determine all the angles in the common "pattern block" shape set based on equilateral triangles. Placing six equilateral triangles so that they share a common vertex (as shown in part a), students can figure out that because the sum of the angles at this vertex is 360°, each angle which shares this vertex must have measure 60°. Because they are congruent, all the angles of the equilateral triangles must have measure 60° (again, to ensure they develop a firm foundation, students can verify these for themselves with a protractor). Because each angle of the regular hexagon (part b) is composed of two angles from equilateral triangles, the hexagon's angles each measure 120°. Similarly, in a pattern block set, two of the smaller angles from tan rhombi compose an equilateral triangle's angle, so each of the smaller rhombus angles has measure 30°.

(*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 24)

Students with an accurate conception of angle can recognize that angle measure is *additive*. As with length, area, and volume, when an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Students can then solve interesting and challenging addition and subtraction problems to find the measurements of unknown angles on a diagram in real world and mathematical problems. For example, they can find the measurements of angles formed a pair of intersecting lines, as illustrated above, or given a diagram showing the measurement of one angle, find the measurement of its complement. They can use a protractor to check, not to check their reasoning, but to ensure that they develop full understanding of the mathematics and mental images for important benchmark angles (e.g., 30º, 45º, 60º, and 90º).

	Determining angle measurements  Students might be asked to determine the measurements of the following angles: $\angle BOD$ $\angle BOF$ $\angle ODE$ $\angle CDE$ $\angle CDJ$ $\angle BHG$ (*Progressions for the CCSSM*, *Geometric Measurement*, CCSS Writing Team, June 2012, page 24)

	 (*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 25)

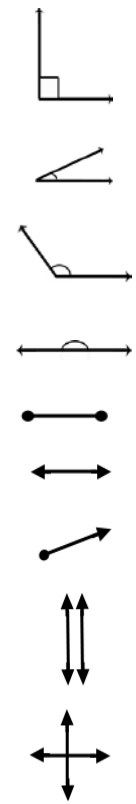
Geometry 4.G

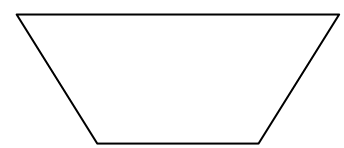
Common Core Cluster

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are**: classify shapes/figures, (properties)-rules about how numbers work, point, line, line segment, ray, angle, vertex/vertices, right angle, acute, obtuse, perpendicular, parallel, right triangle, isosceles triangle, equilateral triangle, scalene triangle, line of symmetry, symmetric figures, two dimensional, regular, irregular**
From previous grades: **polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, cone, cylinder, sphere, kite, parallelogram, examples, non-examples**
[1]The term "**property**" in these standards is reserved for those attributes that indicate a relationship between components of shapes. Thus, "having parallel sides" or "having all sides of equal lengths" are properties. "**Attributes**" and "**features**" are used interchangeably to indicate any characteristic of a shape, including properties, and other defining characteristics (e.g., straight sides) and nondefining characteristics (e.g., "right-side up").
(Progressions for the CCSSM, Geometry, CCSS Writing Team, June 2012, page 3 footnote)

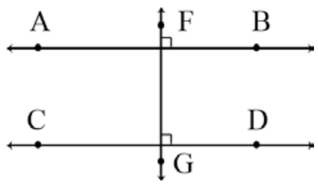
Common Core Standard	**Unpacking** What do these standards mean a child will know and be able to do?

4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	This standard asks students to draw two-dimensional geometric objects and to also identify them in two-dimensional figures. This is the first time that students are exposed to rays, angles, and perpendicular and parallel lines. Examples of points, line segments, lines, angles, parallelism, and perpendicularity can be seen daily. Students may not easily identify lines and rays because they are more abstract. right angle acute angle obtuse angle straight angle segment line ray parallel lines perpendicular lines  Student should be able to use side length to classify triangles as equilateral, equiangular, isosceles, or scalene; and can use angle size to classify them as acute, right, or obtuse. They then learn to cross-classify, for example, naming a shape as a right isosceles triangle. Thus, students develop explicit awareness of and vocabulary for many concepts they have been developing, including points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Such mathematical terms are useful in communicating geometric ideas, but more important is that constructing examples of these concepts, such as drawing angles and triangles that are acute, obtuse, and right, help students form richer concept images connected to verbal definitions. That is, students have more complete and accurate mental images and associated vocabulary for geometric ideas (e.g.,

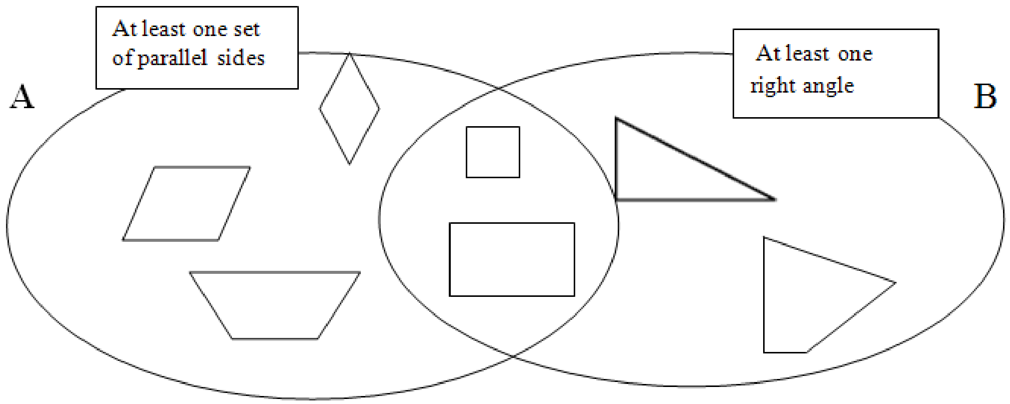
	they understand that angles can be larger than 90 and their concept images for angles include many images of such obtuse angles). Similarly, students see points and lines as abstract objects: Lines are infinite in extent and points have location but no dimension. Grids are made of points and lines and do not end at the edge of the paper. Students also learn to apply these concepts in varied contexts. For example, they learn to represent angles that occur in various contexts as two rays, explicitly including the reference line, e.g., a horizontal or vertical line when considering slope or a "line of sight" in turn contexts. They understand the size of the angle as a rotation of a ray on the reference line to a line depicting slope or as the "line of sight" in computer environments. Analyzing the shapes in order to construct them requires students to explicitly formulate their ideas about the shapes. For instance, what series of commands would produce a square? How many degrees are the angles? What is the measure of the resulting angle? What would be the commands for an equilateral triangle? How many degrees are the angles? What is the measure of the resulting angle? Such experiences help students connect what are often initially isolated ideas about the concept of angle. *(Progressions for the CCSSM, Geometry*, CCSS Writing Team, June 2012, page 14) Example: Draw two different types of quadrilaterals that have two pairs of parallel sides? Is it possible to have an acute right triangle? Justify your reasoning using pictures and words. Example: How many acute, obtuse and right angles are in this shape?  Draw and list the properties of a parallelogram. Draw and list the properties of a rectangle. How are your drawings and lists alike? How are they different? Be ready to share your thinking with the class.
4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	Two-dimensional figures may be classified using different characteristics such as, parallel or perpendicular lines or by angle measurement. Parallel or Perpendicular Lines: Students should become familiar with the concept of parallel and perpendicular lines. Two lines are parallel if they never intersect and are always equidistant. Two lines are perpendicular if they intersect in right angles (90º). Students may use transparencies with lines to arrange two lines in different ways to determine that the 2 lines might intersect in one point or may never intersect. Further investigations may be initiated using geometry software. These types of explorations may lead to a discussion on angles.

A **kite** is a quadrilateral whose four sides can be grouped into two pairs of equal-length sides that are beside (*adjacent to)* each other.

Parallel and perpendicular lines are shown below:



This standard calls for students to sort objects based on parallelism, perpendicularity and angle types.
Example:
Which figure in the Venn diagram below is in the wrong place, explain how do you know?



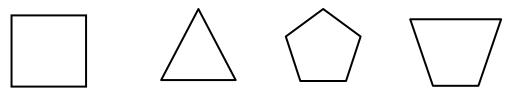
Do you agree with the label on each of the circles in the Venn diagram above? Describe why some shapes fall in the overlapping sections of the circles.

Example:
Draw and name a figure that has two parallel sides and exactly 2 right angles.

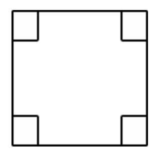
Example:
For each of the following, sketch an example if it is possible. If it is impossible, say so, and explain why or show a counter example.

- A parallelogram with exactly one right angle.
- An isosceles right triangle.
- A rectangle that is *not* a parallelogram. *(impossible)*
- Every square is a quadrilateral.
- Every trapezoid is a parallelogram.

Example:
Identify which of these shapes have perpendicular or parallel sides and justify your selection.

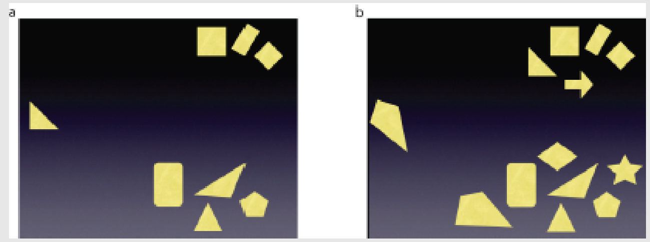


A possible justification that students might give is:
The square has perpendicular lines because the sides meet at a corner, forming right angles.



Angle Measurement:
This expectation is closely connected to 4.MD.5, 4.MD.6, and 4.G.1. Students' experiences with drawing and identifying right, acute, and obtuse angles support them in classifying two-dimensional figures based on specified angle measurements. They use the benchmark angles of 90°, 180°, and 360° to approximate the measurement of angles.
Right triangles can be a category for classification. A right triangle has one right angle. There are different types of right triangles. An isosceles right triangle has two or more congruent sides and a scalene right triangle has no congruent sides.

Guess My Rule

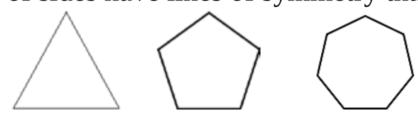


Students can be shown the two groups of shapes in part a and asked "Where does the shape on the left belong?" They might surmise that it belongs with the other triangles at the bottom. When the teacher moves it to the top, students must search for a different rule that fits all the cases.

Later (part b), students may induce the rule: "Shapes with at least one right angle are at the top." Students with rich visual images of right angles and good visualization skills would conclude that the shape at the left (even though it looks vaguely like another one already at the bottom) has one right angle, thus belongs at the top.

(Progressions for the CCSSM, Geometry, CCSS Writing Team, June 2012, page 15)
The notion of congruence ("same size and same shape") may be part of classroom conversation but the concepts of congruence and similarity do **not** appear until middle school.

TEACHER NOTE: In the U.S., the term "trapezoid" may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with *at least* one pair of parallel sides. The exclusive definition states: **A trapezoid is a quadrilateral with *exactly* one pair of parallel sides.** With this definition, a parallelogram is not a trapezoid. North Carolina has adopted the exclusive definition. (*Progressions for the CCSSM: Geometry*, The Common Core Standards Writing Team, June 2012.)

4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	Students need experiences with figures which are symmetrical and non-symmetrical. Figures include both regular and non-regular polygons. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry. This standard only includes line symmetry not rotational symmetry. Example: For each figure, draw all of the lines of symmetry. What pattern do you notice? How many lines of symmetry do you think there would be for regular polygons with 9 and 11 sides. Sketch each figure and check your predictions. Polygons with an odd number of sides have lines of symmetry that go from a midpoint of a side through a vertex. 

Glossary

Table 1 Common addition and subtraction situations[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**[2]
Put Together/ Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[2]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[3]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[4]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2 Common multiplication and division situations[1]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.
[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3 The properties of operations

Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

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